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AN ALGORITHM FOR PORTFOLIO OPTIMIZATION USING VALUE AT RISK

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Dr. Eti MİZRAHİ

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RİSKE MARUZ DEĞER KULLANARAK OLUŞTURULAN PORTFÖY OPTİMİZASYON ALGORİTMASI

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Şansal Başaran

TABLE OF CONTENTS

Page

LIST OF TABLES

Page

LIST OF FIGURES

Page

ABBREVIATIONS

ISE-30	: İstanbul Stock Exchange National 30 Index
VaR	: Value at Risk
AKBNK	: Akbank
AKSA	: Aksa
ARCLK	: Arçelik
ASYAB	: Asya Katılım Bankası
BIMAS	: Bim Mağazaları
DOHOL	: Doğan Holding
ENKAI	: Enka İnşaat
EREGL	: Ereğli Demir Çelik
GARAN	: Garanti Bankası
HALKB	: Halk Bankası
IHLAS	: İhlas Holding
ISCTR	: İş Bankası (C)
KCHOL	: Koç Holding
KRDMD	: Kardemir (D)
MGROS	: Migros
PETKM	: Petkim
SAHOL	: Sabancı Holding
SISE	: Şişe Cam
SNGYO	: Sinpaş GMYO

TCELL	: Turkcell
THYAO	: Türk Hava Yolları
TKFEN	: Tekfen Holding
TOASO	: Tofaş Oto. Fab.
TTRAK	: Türk Traktör
TUPRS	: Tüpraş
VAKBN	: Vakıflar Bankası
YKBNK	: Yapı Kredi Bankası

SUMMARY

Portfolio optimization problem come up with the appearance of the idea investment. It is a process of changing the amount of the financial instruments for fighting the risk. Risk is one of the most important parameters in portfolio optimization. The central part of optimization problem is minimizing risk. There are different ideas about obtaining optimal portfolio. “Traditional Portfolio Theory” and “ Modern Portfolio Theory” are two important theories which aims to reduce the risk of the portfolio. Harry Markowitz who is known as the father of Modern Portfolio Theory, publised an article named *“Portfolio Selection”* in 1952 and changed the world of investing forever.

In this study we determine two optimal portfolios composition using two main optimal portfolio idea. Traditional Portfolio Theory supports increasing number of asset can reduce the risk. Using this idea we composed a portfolio which contains all assets which are trading in ISE-30. These assets are equally weighted in portfolio. We calculate return and variance of this portfolio. Then we build up another portfolio using Mean Variance Method which is the basis of the Modern Portfolio Theory. Optimal portfolio which has same expected level with equally weighted portfolio, is determined by using Mean Variance Method. In this study we use Value at Risk as a risk measure. After we determine two optimal portfolio we measure VaR of these portfolios.

As a conclusion we determine which portfolio performs better than the other comparing their VaR calculation results. This is also a comparison between Traditional Portfolio Theory and Modern Portfolio Theory.

ÖZET

Yatırım fikrinin ortaya çıkmasıyla birlikte yatırımcıların en çok merak ettikleri konu ellerinde bulunan sermayelerini en optimimum şekilde nasıl dağıtabilecekleri olmuştur. Portföy optimizasyon sürecinde, yatırımcının amacı portföyünün riskini minimize etmektir. Riski minimize edebilmek için finans dünyasında iki temel teori kabul görmüştür. Bunlardan biri Geleneksel Portföy Teorisi diğeri ise Modern Portföy Teorisidir. 1950lere kadar Geleneksel Portföy Teorisi finans dünyasında kabul görmüştür. Harry Markowitz'in 1952 yılında yayınladığı, Modern Portföy Teorisinin temellerini oluşturan "Portfolio Selection" isimli yatırım dünyasında büyük ses getirdi.

Bu çalışmada bu iki temel portföy yatırım teorisi kulanılarak optimal portföyler oluşturulmuştur. Geleneksel Portföy Teorisi portföy içerisindeki varlıkların sayısının arttırılmasıyla portföy riskinin düşeceğini savunur. Bu düşünce doğrultusunda biz de IMKB-30 bünyesinde işlem gören ve elimizde verisi bulunan 27 adet hisse senedinin tamamının yer aldığı bir portföy oluşturduk. Bu hisse senetleri portföy içerisinde eşit ağırlığa sahip olacak şekilde yer aldı. Bunun ardından Modern Portföy Teorisinin temelini oluşturan Mean Variance optimizasyon yöntemini kullanarak yeni bir portföy oluşturduk. Bu portföyün getirisi, karşılaştırma yapılabilmesi amacıyla eşit ağırlıklı portföyün getirisine eşit seçilmiştir. İki farklı portföyü belirledikten sonra bu iki portföyün risk hesaplamaları Riske Maruz Değer(VaR) yardımıyla yapıldı. Riske Maruz Değer, portföyün sahip plduğu bütün riskleri tek bir veri olarak bize veren bir risk ölçüm metododur.

Çalışmanın sonunda elde edilen iki portföyün performansını karşılaştırmak amacıyla Riske Maruz Değer sonuçlarını gözlemledik. Hangi portföyün aynı getiri seviyesinde daha düşük riske sahip olduğuna Riske Maruz Değer verilerini karşılaştırarak kara verdik. Bu karşılaştırma aynı zamanda Geleneksel Portföy Teorisi ile Modern Portföy Teorisinin karşılaştırması olmuştur.

1. INTRODUCTION

Portfolios are an appropriate mix or collection of investments held by an institution or individual. The aim of building a portfolio is allocating risk on various assets. There are two theories which defend investing more than one asset to minimize risk; “Traditional Portfolio Theory”, “Modern Portfolio Theory”.

Traditional Portfolio Theory accept that risk can be reduced if the number of assets increases and this theory had common use until 1950s. But then it was argued that, the investement of a big number of assets include also assets with lower return which will reduce also the portofolio return. So a new thought come up which called Modern Portfolio Theory. Modern Portfolio Theory aims to allocate assets by maximising the expected return at the lowest possible risk.

Modern Portfolio Theory developed by Harry Markowitz in the early 1950s. This theory provides a mathematical framework in which investors can minimize risk and maximize expected return. The central part of the theory is that diversifying holdings can reduce risk and that returns are a function of expected risk. In an other word it is an investement approach which based on constructing optimal portfolio with maximum expected return for a given level of risk.

Risk measurement of a portfolio may be helpful for investors to make quick decisions before the value of portfolio become to decrease. Value at Risk (VaR) is a widely used risk measure of the risk of loss on a specific portfolio of financial assets. Specifically value at risk is a measure of losses due to “normal” market movements. Losses greater than VaR are suffered only with a specified probability. Value at risk aggregates all of the risks in a portfolio into a single number so it is a simply way to describe the magnitude of the likely losses on the portfolio.

In this work we determine an optimal portfolio in ISE-30 using Markowitz Mean-Variance method and Traditional asset allocation then we measure risk of the optimal portfolio with Historical simulation VaR method. At last we compare these two optimal portfolios obtained by these two different methods.

In section 2, we give theoritical information about two main portfolio selection; Traditional Portfolio Theory and Modern Portfolio Theory. In section 3 we define a risk measure, Value at Risk which we use for measuring risk of two optimal portfolios. In section 4, we apply Mean-Variance Method in ISE-30 and we determine equal weighted portfolio. We do a comparison between two optimal portfolios' VaR in section 5.

2. PORTFOLIO THEORY

2.1. Definition of Portfolio

Portfolio is an financial term which defines investing in a group of assets instead of one asset. Portfolios can involve different types of financial assets such as stocks, bonds, cash, mutual funds. Investors prefer to invest their money in more than one asset. Because we don’t want to take all the risk of one asset and we prefer sharing the it.

If we have a single or a small number of assets in our portfolio, there appears unsystemic risk. Investing in various assets will eliminate this unsytemic risk and the portfolio will have a minimum level of risk, systemic risk. Systemic risk affects almost all assets so a portion of risk which cannot be eliminated exist in portfolios[1]. So it is possible to reduce the total amount of risk in a portfolio by choosing several assets.

While investors building a portfolio first they must think carefully about their preferences. They must determine expected return and risk aversion first, then they have to decide which amount of asset invested in portfolio[2].

2.2. Portfolio Theories

There are two main theories about reducing the total risk of the portfolio. Traditional Portfolio Theory had had common use until 1950s. This theory states the risk of a portfolio can reduce just increasing the number of assets. The other theory is called Modern Portfolio Theory and it aims to decrease risk of portfolio considering means of returns, the variance of returns and correlations between returns on assets[3].

2.2.1. Traditional Portfolio Theory

Traditional Portfolio Theory supports great diversification of investments can reduce the risk of the portfolio and it doesn’t take care the correlations between investments.

The returns of the investments in the portfolio can’t move in the same direction so portfolio risk is smaller than one asset. If the number of investmens is incresed the risk of the portfolio reduces acceptable risk levels. This approach calls “Putting all eggs in one basket”[4].

2.2.2. Modern Portfolio Theory

In Traditional Portfolio theory investor adds assets in the portfolio not looking their returns, so low returned assets can take place in the portfolio and this causes a new problem. Adding assets not looking relations between assets reduce the total risk of the portfolio but it also reduces total return of the portfolio. For solving this problem Harry Markowitz who is father of modern portfolio theory, published an article in 1952, named “Portfolio Selection” and a book in 1959 “Portfolio Selection: Efficient Diversification of Investments”. In these publications, he set up a new theory for portfolio selection. Markowitz formulated portfolio problem as a choice of the mean and variance of a portfolio of assets.

The theory clarifies that the assets could not be selected only on characteristics of unique asset. An investor had to consider how each security co-moved with all other securitites. taking these co-movements into account resulted in an ability to construct a portfolio that had the same expected return and less risk than a portfolio constructed by ignoring the interactions between securities[5]. Adding assets whose correlations are smaller than 1 in a portfolio reduces risk and it prevents to decrease the return of portfolio.[6].

The Markowitz model is based on several assumptions regarding investor behavior[7]:

- Investors consider each investment alternative as being presented by a probability distribution of expected returns over some holding period.

- Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.

- Investors estimate the risk of the portfolio on the basis of the variability of expected returns.

- Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and the expected variance of returns only.

- For a given risk level, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk

2.3. Markowitz Mean-Variance Approach

The mean-variance framework for portfolio selection, developed by Markowitz (1952), continues to be the most popular method for portfolio construction[8].

Mean variance portfolio theory developed to find a solution the concerns on return distributions. An investor have to estimate the mean, return and variance of return for each asset which can be take place in the portfolio[5]. If investor have the estimitations of returns, volatilities, and correlations of a set of investments, it is possible to perform an optimization that results in the risk/return or mean-variance efficient frontier. This frontier is efficient because it gives the maximum expected return for that level of risk or minimum risk for that level of expected return[9].

Assumptions of mean variance analysis[10]:

1. Investors' preferences can be expressed with a mean-variance utility function. That is, they are only concerned with the expected return and the variance of portfolios over a particular period.

2. Financial markets are frictionless, i.e.

- Investors take prices as given
- Assets are infinitely divisible
- No transaction costs or taxes

3. One period investment horizon

2.4. Risk and Return

2.4.1. Portfolio Return

The return on a portfolio of assets is simply a weighted average of the return on the individual assets. We can calculate the return of an asset dividing the difference between two preiod of stock price to first stock price.

The rate of return of an individual asset can be calculate as an below[11],

$$r = \frac{P_t - P_{t-1}}{P_{t-1}} \qquad (2.4.1.1)$$

r = Rate of return of an asset

P_t = Price of the asset at time *t*.

P_{t-1} = Price of the asset at time *t-1*.

The weight applied to each return is the fraction of portfolio invested in that asset. Suppose that there are n securities, let r_i be the return at time t invested in security *i* and let w_i be the relative amount invested in security *i*, then the return of the portfolio is[6],

$$r_p = w_1 r_1 + w_2 r_2 + w_3 r_3 + w_n r_n \qquad (2.4.1.2)$$

$$r_p = \sum_{i=1}^{n} w_i r_i \qquad (2.4.1.3)$$

r_p = The return of the portfolio

w_i = Amount invested in security i

r_i = The return invested in security i

n = The number of assets in the portfolio

2.4.2. Portfolio Risk

Risk of a portfolio depends on relations between assets so we cannot calculate it like we calculate return of the portfolio. So while we are measuring the risk we have to calculate covariance which defines the relation between assets and we have to know correlation coefficeient. Correlation coefficient can calculate as,

$$\rho_{i,k} = \frac{\mathrm{cov}(r_i, r_k)}{\sigma_i \sigma_k} \qquad (2.4.2.1)$$

$\rho_{i,k}$ = Correlation coefficient between the return of the asset i and asset k

Cov(r_i, r_k) = Covariance between the return of asset i and asset k

σ_i = Standart deviation of asset i

σ_k = Standart deviation of asset k

Markowitz theory an optimal set of weights is one in which the portfolio achieves an acceptable baseline expected rate of return with minimal volatility. Here the variance of the rate of return of an instrument is taken as a surrogate for its volatility.

We can calculate portfolio risk which contain more than two assets as below,

$$\sigma_p^2 = \sum_{i=1}^{n} w_i\,\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{\substack{k=1\\k\neq i}}^{n} w_i w_k \operatorname{cov}(r_i, r_k) \qquad (2.4.2.2)$$

$$\begin{aligned}\sigma_p^2 = {} & w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + \ldots + w_n^2\sigma_n^2 + 2w_1w_2\operatorname{cov}(r_1, r_2) \\ & + 2w_1w_3\operatorname{cov}(r_1, r_3) + 2w_2w_3\operatorname{cov}\left(r_{2,}r_3\right) + \cdots \\ & + 2w_{n-1}w_n\operatorname{cov}(r_{n-1}r_n) \qquad (2.4.2.3)\end{aligned}$$

We can also write that formula as below,

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{k=1}^{n} w_i w_k \operatorname{cov}(r_i, r_k) \qquad (2.4.2.4)$$

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{k=1}^{n} w_i w_k\ \rho_{i,k}\sigma_i\sigma_k \qquad (2.4.2.5)$$

If we want to find variance or standart devation of a portfolio that contains N assets we have to build up a covariance matrice which has N row and N column. We find variance using covariance matrice and if we take square root of variance we find portfolio risk.

Table 2.1. Covariance Matrice

	1	2	3	…	
1	$w_1 w_1 \rho_{1,1} \sigma_1 \sigma_1$ or $w_1 w_1 \text{cov}(r_1, r_1)$	$w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2$ or $w_1 w_2 \text{cov}(r_1, r_2)$	$w_1 w_3 \rho_{1,3} \sigma_1 \sigma_3$ or $w_1 w_3 \text{cov}(r_1, r_3)$	⋮	
2	$w_2 w_1 \rho_{2,1} \sigma_2 \sigma_1$ or $w_2 w_1 \text{cov}(r_2, r_1)$			⋮	
3	$w_3 w_1 \rho_{3,1} \sigma_3 \sigma_1$ or $w_3 w_1 \text{cov}(r_3, r_1)$			⋮	
:	:			:	
N	$w_N w_1 \rho_{N,1} \sigma_N \sigma_1$ or $w_N w_1 \text{cov}(r_N, r_1)$			⋮	$w_N w_N \rho_{N,N} \sigma_N \sigma_N$ or $w_N w_N \text{cov}(r_N, r_N)$

$$\sigma_p^2 = [w_1 \quad w_2 \quad \dots \quad w_N] \begin{bmatrix} \text{cov}(r_1, r_1) & \text{cov}(r_1, r_2) & \dots & \text{cov}(r_1, r_N) \\ \text{cov}(r_2, r_1) & \text{cov}(r_2, r_2) & \dots & \text{cov}(r_2, r_N) \\ . & . & \dots & . \\ . & . & \dots & . \\ \text{cov}(r_N, r_1) & \text{cov}(r_N, r_2) & \dots & \text{cov}(r_N, r_N) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ . \\ . \\ w_N \end{bmatrix} \quad (2.4.2.6)$$

Then we can write covariance matrice of portfolio which contains N assets as,

$$Q = \begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & \dots & . \\ 0 & \sigma_3 & 0 & \dots & . \\ . & . & . & \dots & . \\ 0 & . & 0 & \dots & \sigma_N \end{bmatrix} \begin{bmatrix} \rho_{1,1} & \rho_{1,2} & \dots & \rho_{1,N} \\ \rho_{2,1} & \rho_{2,2} & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ \rho_{N,1} & . & \dots & \rho_{N,N} \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & . & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ 0 & . & . & \dots & \sigma_N \end{bmatrix} \quad (2.4.2.7)$$

According to these equations the variance of portfolio is,

$$\sigma_p^2 = wQw^T \quad (2.4.2.8)$$

σ_p^2 = Variance of the portfolio w = Vector of amounts

Q = Covariance matrice

2.5. Efficient Frontier

The risk/return trade off has been central principle of portfolio management since the seminal work of Markowitz(1952).Higher returns can only be achieved at the great level of risk. This thought is the concept of the efficient frontier[13]. Efficient frontier simply defines maximum return that can be achieved given level of risk or minimum risk that must be incurred to earn a given return.

Every possible combination of risky assets, can be plotted in risk-expected return space, and the collection of all such possible portfolios defines a region in this space. The upward-sloped part of the left boundary of this region, a curve, is then called the efficient frontier.

Figure 2.1. shows investors the entire investment opportunity set, which is the set of all attainable combinations of risk and return. These portfolios are composed by different proportionsn of asset A and B. The curve passing through A and B shows the risk-return combinations of all the portfolios that can be formed by combining those two assets. Investors desire portfolios that lie to the northwest in Figure 2.1. These are portfolios with high expected returns and low volatility[4].

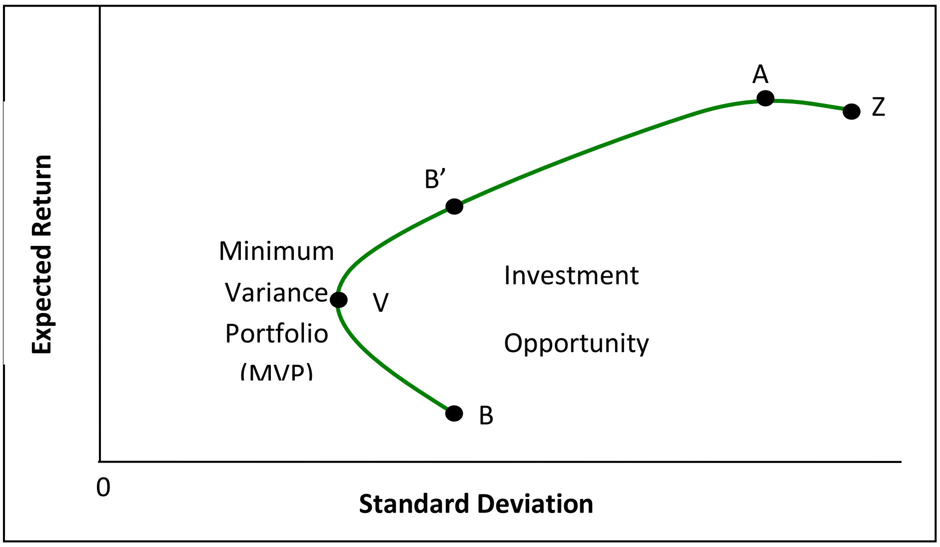

Figure 2.1 Investment opportunity set for asset A and asset B

The best choice among the portfolios on the upward sloping portion of the frontier curve is not obvious, because in this region higher expected return means higher risk. The best choice will depend on the investor's willingness to trade off risk against expected return.

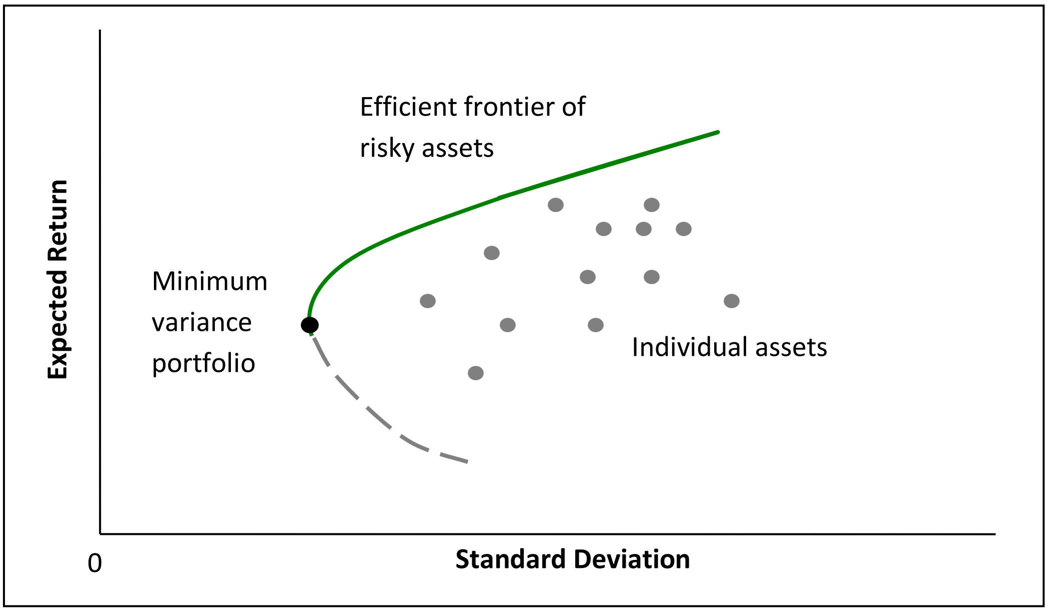

Figure 2.2 The efficient frontier of risky assets and individual assets

2.6. Mean Variance Optimization Model

Markowitz (1952) used variance as the measure of the risk and used mean return as the expected return in the mean variance model. The Mean Variance Model aims to find weight of assets to minimize portfolio risk at a level of required rate of return[14]. This model is quadratic programming model. The mathematical model is as follows;

$$Minimize \sum_{i=1}^{N}\sum_{k=1}^{N} w_i w_k \operatorname{cov}(r_i\,, r_k) \tag{2.6.1}$$

subject to

$$\sum_{j=1}^{N} w_j r_j = R \tag{2.6.2}$$

$$\sum_{j=1}^{N} w_j = 1 \tag{2.6.3}$$

$$w_j \geq 0 \qquad j = 1,2, \dots, N \qquad (2.6.4)$$

where

w_i = The fraction of asset i

r_i = Expected return of asset i

Cov(r_i,r_k) = Covariance of r_i and r_k

R = Required level of return for the portfolio

N = Number of assets

The Markowitz model is popular because of its simplicity. This model consists of two summary statistics which are mean and variance. Moreover, it is easy to construct the efficient frontier with the combination of return and risk.

In Markowitz Mean Variance Model there are two main constraint,

$$\sum_{j=1}^{N} w_j r_j = R$$

Constraint shows weight of return of the assets which construct the portfolio must be equal a specified level of return.

$$\sum_{j=1}^{N} w_j = 1$$

Constraint shows the weight of assets have to be equal 1 so we make all our capital invest to assets, no other investment.

$w_j \geq 0$ j = 1,2, ... ,N

This shows the weight of asset must be positive so we avoid short sale problem.

Solution of this problem which minimizes variance and provides given constraint gives us optimal portfolio for a given level of return.

3. VALUE AT RISK

3.1. Definition of Value At Risk

Risk can be defined as the degree of uncertinty of future net returns. This uncertinty takes many forms. We can classify risks as below,

• Credit risk estimates the potential loss because of the inability of a counterparty to meet its
obligations

• Operational risk results from errors that can be made in instructing payments or settling transactions

• Liquidity risk is reflected in the inability of a firm to fund its illiquid assets

• Market risk involves the uncertainty of future earnings resulting from changes in market conditions, (e.g., prices of assets, interest rates)

Value at Risk has, in the past few years, become a key component of the management of market risk for many financial institutions. It is used most often by investment banks to avoid the potential loss of their portfolio. This potential loss can be caused from adverse market movements over a specified period[14].

In the mean-variance portfolio optimization model, Markowitz uses the portfolio variance as the risk measure. The variance is a measure of statistical distribution. It gives us possible squared distance from expected value. It treats both upside and downside payoffs symmetrically. However, while most investors will be disturbed by a drop in prices, most investors will not mind an increase in price.

Value at Risk is one of the alternative risk measure. Value at Risk is defined as the difference between the corresponding percentile of the profit and loss distribution and the current value of the portfolio[15]. "Value at risk can be defined as the maximum expected loss on a portfolio over a given horizon period, at a given confidence level". The horizon period and the confidence level are two arbitrary parameters of VaR concept. Horizon period must be a day, a week, etc; and confidence level might be 90%, 95%, 99%. For example if we have daily period data and 95% confidence level then VaR would be maximum expected loss over a day, at the 95% confidence level[16]. In Figure 3.1. we can see the example of one day VaR of $10X at the 95% cofidence level.

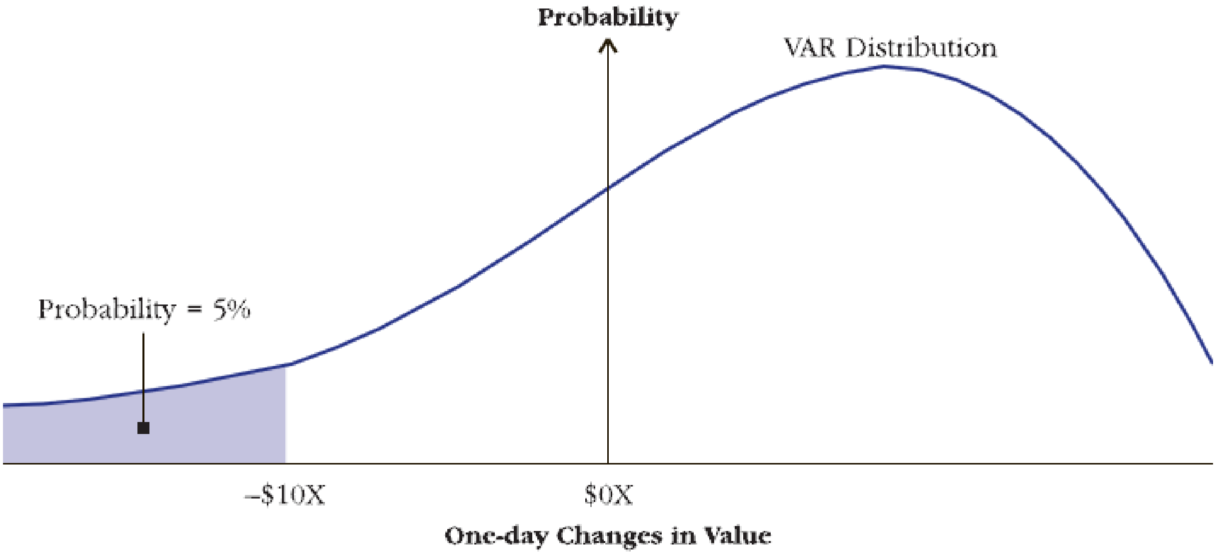

Figure 3.1. Example of one day VaR

Value at Risk is powefull and still simple tool in risk management so it provides a lot of advantages to portfolio managers. We can put these advantages in order as below[16];

1. It can address all types of risks in a single framework,

2. It facilitates decisions on the allocation of capital and gives an indication of the total capital requirement

3. It can be used to set risk limits for individual traders, divisions, and the entire company; it

facilitates performance measurement and can be used as the basis for performance-related pay finally, it can help to decide which risks to reduce, if necessary

4. It also aids in detecting any trends in the behavior of individuals, divisions, or the company as a whole.

3.2. Computing VaR

VaR summarizes the expected maximum loss over a target horizon within a given confidence level. We take quantitive factors, horizon and confidence level as given. We can follow these steps while computing VaR;

- Determine the value of current portfolio
- Measure the variability of risk factors
- Set the holding period
- Set the confidence level
- Determine the worst expected loos using all the preceding information

We decided the steps of constructing VaR so first we have to determine the parameters of VaR then we measure VaR.

3.3. Determining VaR Parameters

3.3.1. Holding Period

VaR is depandant on the choice of the holding period so we should consider how holding period affects VaR. This Relation is illustrated in Figure 3.2. The Figure 3.2 plots VaR at the 95% confidence level against a holding period that varies from 1 to 100 days.

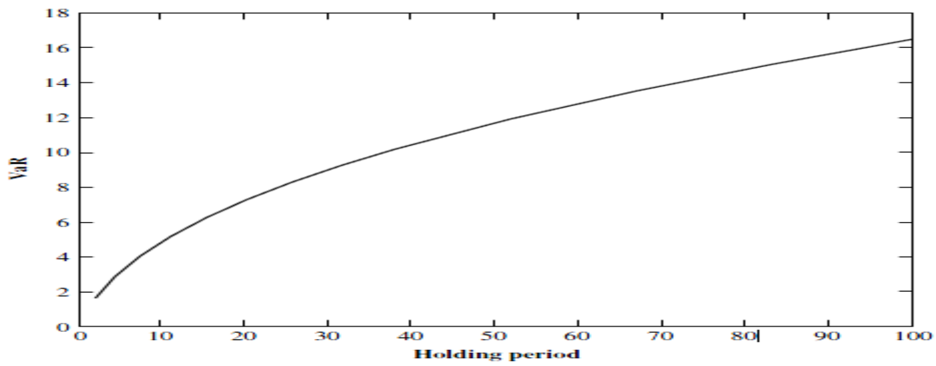

Figure 3.2. VaR and Holding Period

VaR calculations are always initially done on a holding period of 1 day, since this provides the maximum amount of historical information with which to estimate parameters. In the case where portfolio returns assumed, VaR rises with the square root of the holding period. So the holding period generally is selected 1 day by institutions[21].

Basel Committee determines three important standard for holding period and the number of datas while calculating VaR[22];

1. "Value-at-risk" must be computed on a daily basis.

2. In calculating value-at-risk, an instantaneous price shock equivalent to a 10 day movement in prices is to be used, i.e., the minimum "holding period" will be ten trading days.

3. The choice of historical observation period (sample period) for calculating valueat-risk will be constrained to a minimum length of one year.

3.3.2. Confidence Level

VaR is also depandant on confidence level so VaR changes when confidence level changes. The relation between VaR and confidence level is shown in Figure 3.3. In Figure 3.3 VaR is determined by the cut-off between the top 99% and the bottom 1% of observations. So we deal with a 1% tail rather than the earlier 5% tail. “The higher confidence level means a smaller tail, a cut-off point further to the left and, therefore, a higher VaR.”[17]. In general if other things are equal VaR rises when confidence level rises.

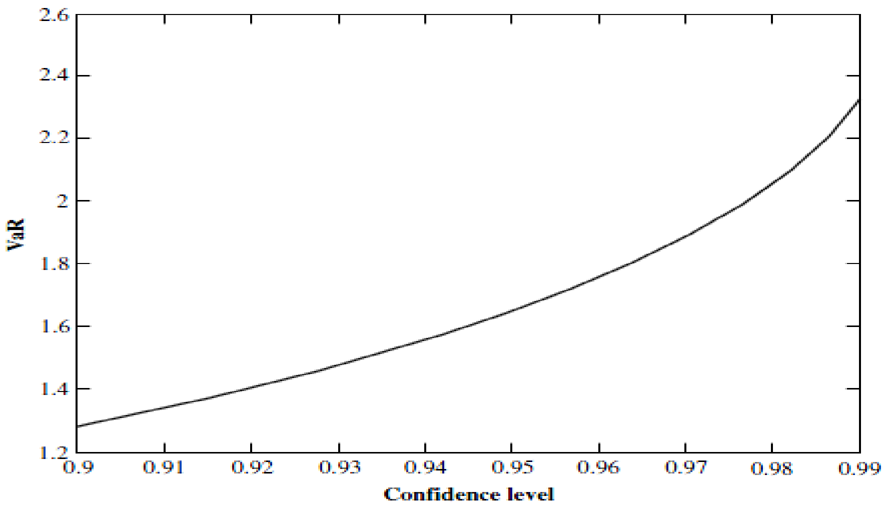

Figure 3.3. VaR and Confidence Level

Confidence level is selected at low level because of getting a reasonable proportion of excess-loss observations. If we estimate VaR for reporting and other purposes, we use confidence level as between 95-99% which is used by most of the institutions[17]. Basel Committe determined confidence level as 99% percent in 1996[18].

3.4. VaR Calculation Methods

Value at Risk calculation methods have different approaches and different assumptions. Multiple VaR methodologies are available and each has its own benefits and drawbacks.

There are three basic approaches that are used to compute Value at Risk. These are the VaR calculation methods which are also suggested by Basel Committee;

- Variance-Covariance Method
- Historical Simulation Method
- Monte Carlo Simulation Method

3.4.1. Variance-Covariance

Value at Risk measure the probability that the value of the portfolio will drop below a value in a time period. From this thought if we want to calculate VaR we have to derive a probability distribution of potential loss.

Variance-Covariance Method assumes the returns on risk factors are normally distributed, the correlations between risk factors are constant and the delta (or price sensitivity to changes in a risk factor) of each portfolio constituent is constant. Variance-Covariance Method can be called as a direct application of Markowitz portfolio theory. Because this method measures the risk of the portfolio as standart deviation based on variances and covariances of the returns of the assets.

While calculating VaR of portfolio with Variance-Covariance Method, portfolio volatility have to be calculated. We have to calculate variance and covariances of assets for determining volatility of portfolio. VaR can be determined as,

$$V\mathrm{aR}^{p}_{proportional} = (z_\alpha)\sigma_p\left(\sqrt{t}\right) \quad (3.5.1.1)$$

$$\sigma_p^2 = wQw^T \quad (3.5.1.2)$$

where Q is variance-covariance matrice given by,

$$Q = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho_{1,2} & \dots & \sigma_1\sigma_K\rho_{1,k} \\ \sigma_2\sigma_1\rho_{1,2} & \sigma_2^2 & \dots & \sigma_2\sigma_k\rho_{2,k} \\ . & . & \dots & . \\ . & . & \dots & . \\ \sigma_k\sigma_1\rho_{1,k} & \sigma_k\sigma_2\rho_{2,k} & \dots & \sigma_k^2 \end{bmatrix} \tag{3.5.1.3}$$

σ_i is the standard deviation(volatility) of $\rho_{i,j}$ is the correlation coefficient between the returns of r_i and r_j.

$$V\mathrm{aR}^{p}_{proportional} = (z_\alpha)\sqrt{wQw^T}\,(\sqrt{t}) \tag{3.5.1.4}$$

If we add portfolio wealth to formula we can get absolute VaR;

$$V\mathrm{aR(absolute)} = P_0(z_\alpha)\sqrt{wQw^T}(\sqrt{t}) \tag{3.5.1.5}$$

where z_α is the inverse of the cumulative normal distribution function. In Figure 3.4 there is an exapmle of normally distributed VaR.

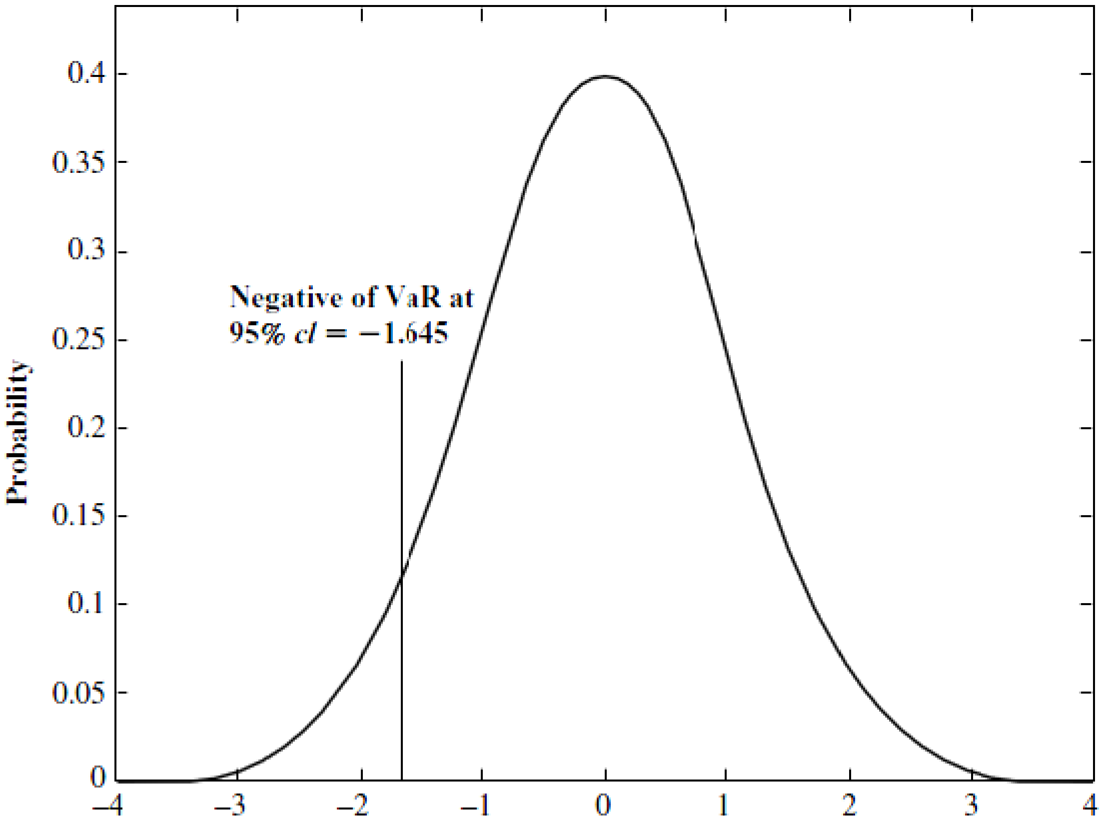

Figure 3.4. Normally Distributed VaR

Variance Covariance method is the simpliest way to calculate VaR. The input data is limited and it has no simulation so we can compute VaR wasting minimum time. Its simplicity is also its disadvantage. Because it assumes not only that historical returns follow a normal distribution it also assumes the changes in price of assets that included in portfolio follow normal distribution and it has a minimum chance in reality. This method has also difficulties while calculating VaR with non-linear payoffs like options[19].

3.4.2. Historical Simulation

A different approach for VaR assessment is called Historical Simulation. Historical simulations represent the simplest way of estimating the Value at Risk for many portfolios. This technique is nonparametric and does not require any distributional assumptions. This is because Historical Simulation uses only the empirical distribution of the portfolio returns[20].

Historical Simulation Method assumes that current portfolio was held in the past so our portfolio meets past changes in risk factors over a period. It means that hypothetical changes in the portfolio constructed on the basis of real past changes in risk factors. This methods doesn't talk about the probabilty distributions of returns and it assumes that historical datas also proceed in the future. Because of this speciality of this method it is called as non-parametric VaR calculation method.

We can calculate VaR of a portfolio with Historical Simulation Method in four basic steps;

1. We calculate the changes in price of the assets which included in portfolio to calculate the portfolio wealth.

2. We multiply wieghts of assets that included in portfolio by past changes in individual assets after this we find past changes in portfolio price. We can show these changes as[20];

$$r_t^p = \sum_{i=1}^{n} w_i r_{i,t} \qquad t = 0,1,2,3,\dots,T \qquad (3.5.2.1)$$

where r_t^p is the return of portfolio at time t, $r_{i,t}$ is the return of the asset i at time t and n is the number of assets.

3. These values arrange from upper to lower

4. At last we find VaR for given confidence level.

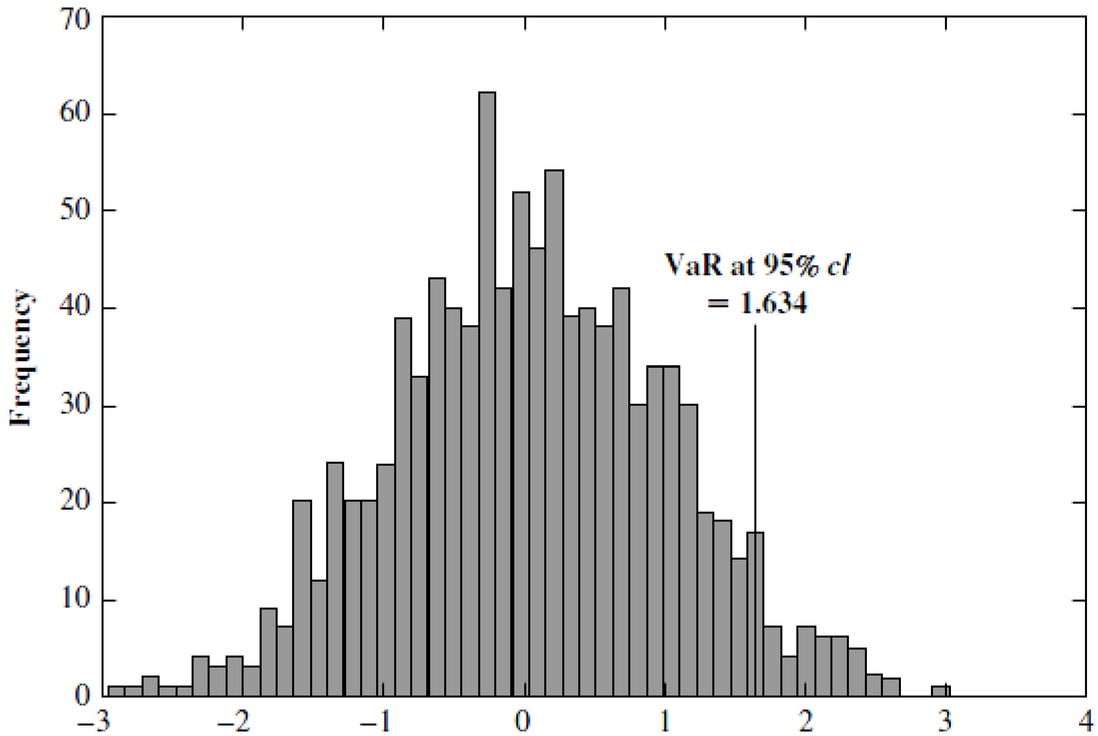

Figure 3.5. Historical Simulation VaR

Computing VaR using the Historical Simulations methodology has several advantages. First, there is no need to formulate any assumption about the return distribution of the assets in the portfolio. Second there is no need to determine volatilities and correlations between asset and this provides simplicity in computing. The fat tails of the distribution and other extreme events are captured as long as they are contained in the dataset. The Historical Simulations VaR methodology may be very intuitive and easy to understand, but it still has a few drawbacks. For instance, if we run a Historical Simulations VaR in a bull market, VaR may be underestimated. In order to increase the Historical Simulation VaR we can increase the number of observations so we have more information where today prices would go[20].

3.4.3 Monte Carlo Simulation

The last of the three main methedologies to calculate Value at Risk is Monte Carlo Simulation Method. The Monte Carlo simulation method involves randomly generating scenarios based on parameters obtained from historical data. After generating these scenarios, we then proceed to calculate the profitt and loss. Generating a lot of random numbers(from a few hundred to a few millions depending on the problem) gives a great indication of output.

Monte Carlo Simulation has great similarities with Historical Simulation Method. It is an extension of Historical Simulation. The main difference between these two simulation methods is; Historical Simulation uses real historical data of returns(or prices) and assumes it wil re-occur in the future. However in Monte Carlo Simulation we generate random numbers to estimate the returns or prices of the assets that included in portfolio. We can apply Monte Carlo Simulation Method in five steps [21];

1. " Determine the length T of the analysis horizon and divide it equally into a large number N of small time increments Δt (i.e. $\Delta t = T/N$)"

2. " Draw a random number from a random number generator and update the price of the asset at the end of the first time increment"

3. " Repeat Step 2 until reaching the end of the analysis horizon T by walking along the N time intervals"

4. "Repeat Steps 2 and 3 a large number M of times to generate M different paths for the stock over T"

5. "Rank the M terminal stock prices from the smallest to the largest, read the simulated value in this series that corresponds to the desired $(1-\alpha)\%$ confidence level (95% or 99% generally) and deduce the relevant VaR"

The main benefit of Monte Carlo Simulations is that they can model instruments with non-linear and path-dependent payoff functions, especially complex derivatives. They have no problem while they are dealing with multiple risk factors, correlations and fat tails. Monte Carlo Simulations VaR is not affected extreme events as much as other VaR calculation methods. The main disadvantage of Monte Carlo simulations is slowness. Because there are huge number of calculations and it takes time and also there must be a powerfull computer to handle these calculations. If we have a portfolio of 1,000 assets and want to run 1,000 simulations on each asset, we will need to run 1 million simulations[17].

4. DETERMINATION OF EQUAL WEIGHTED PORTFOLIO AND DETERMINATION OF OPTIMAL PORTFOLIO IN ISE-30 by USING MEAN-VARIANCE MODEL

4.1. The Purpose of the Research

All the rational investors want to know the risk and return of the portfolio they invest. They want to gain high level return with minimum risk. Because of this they would invest their money a lot number of investment tool rather than one. As we mentioned before first Traditional Portfolio Theory were used which supports just increasing the number of asset diminishes the risk of the portfolio. After 1950s Modern Portfolio Theory appeared. It supports that investor must also take care of relations of assets they invest to avoid low portfolio return levels. Beside decraesing the risk of the portfolio measuring maximum expected loss has become a requirement last years. If we know the expected level of maximum loss then we have an idea about portfolio performance and we can make decisions about our investment.

In this research we determine a portfolio which considers the principle of Traditional Portfolio Theory. We put all of ISE-30 assets in our portfolio and we build up equally weighted portfolio. Then we build up a portfolio. While building this portfolio we consider the relation between asset. Then we compare these two portfolios and decide which we invest.

4.2 Data Collection

In this research we use 27 assets which are trading at ISE-30. We have closing prices between 22.3.2012-1.4.2008. It is a sample which has 1000 datas between these dates. We have to calculate return of portfolios so we use closing price of assets and procure daily returns of assets as using the formula (2.4.1.1).

4.3. Application of Equal Weighted Portfolio

Traditional Portfolio Theory supports diversification of asset can reduce the risk of the portfolio. This theory isn't interseted in correlations between the assets. Using this theory we determine an application in ISE-30. We take all 27 assets which we have as data, in a portfolio. Each asset have equal weight in portfolio and it is found as $\frac{1}{27} \times 100 = 3{,}7\%$. Then we calculate average of the returns and standard deviation of assets indivudally. In Table 4.1. these values are shown;

Table 4.1. Standard deviation and average return of assets

Stock Symbol	Weight	Std Deviation	Means	Stock Symbol	weight	Std Deviation	Means
AKBNK	0,037037	0,02961	0,001002	**MGROS**	0,037037	0,02869	0,000953
AKSA	0,037037	0,024178	0,001642	**PETKM**	0,037037	0,022676	0,000714
ARCLK	0,037037	0,027719	0,001173	**SAHOL**	0,037037	0,027355	0,00108
ASYAB	0,037037	0,028028	-9,50E05	**SISE**	0,037037	0,02494	0,00127
BIMAS	0,037037	0,027966	0,00179	**SNGYO**	0,037037	0,028462	0,000713
DOHOL	0,037037	0,031932	0,000476	**TCELL**	0,037037	0,022528	0,00017
ENKAI	0,037037	0,026644	0,000198	**THYAO**	0,037037	0,026478	0,001454
EREGL	0,037037	0,026515	0,000428	**TKFEN**	0,037037	0,028047	0,000501
GARAN	0,037037	0,028974	0,001028	**TOASO**	0,037037	0,030172	0,001128
HALKB	0,037037	0,030244	0,001113	**TTRAK**	0,037037	0,028259	0,001657
IHLAS	0,037037	0,032818	0,001294	**TUPRS**	0,037037	0,025288	0,001148
ISCTR	0,037037	0,026667	0,000767	**VAKBN**	0,037037	0,028573	0,000721
KCHOL	0,037037	0,025753	0,001319	**YKBNK**	0,037037	0,028261	0,000823
KRDMD	0,037037	0,026771	0,000666				

We can calculate the expected return of the portfolio using the equation (2.4.1.3). It is difficult to calculate these value without help of software so we use Microsoft Excel for calculations. We can also calculate variance of portfolio using (2.4.2.4) or (2.4.2.5). We calculate Expected return and variance of equal weighted portfolio in Mictosoft Excel. The results can be seen in Table 4.2.

Table 4.2. Results of Equal weighted portfolio

Expected Return(%)	0,093
Variance	0,000355
Standard Deviation	0,018838

4.4. Application of Mean Variance Optimal Portfolio

In this part we determine an optimal portfolio using Mean Variance Method. We use assets which trade in ISE-30. We build up an optimal portfolio which has the same expected return as equal weighted portfolio. We use Microsoft Excel and its optimization tool Solver to optimize our portfolio.

As we mentioned in Section 2 we can minimize variance of the portfolio using (2.6.1). While finding optimal portfolio we use some constraints. (2.6.2) shows the expected return of the portfolio must be equal a specified level of return. In our model we determine this level as expected return of equal weighted portfolio which is 0,00093. We use (2.6.3) as an another constraint which shows the sum of weights of the assets have to equal 1. The last constraint is (2.6.4) and it defines weight of assets must be greater than 0 to avoid short selling problem. We find optimal portfolio using these as a constraint in Microsoft Excel Solver. In Table 4.3. there are weights of the assets which are invested in optimal portfolio.

Table 4.3. Weights of Mean Variance Optimal Portfolio Assets **(%)**

Stock Symbol	Weight	Stock Symbol	Weight
AKBNK	0	**MGROS**	14,8886
AKSA	11,59573	**PETKM**	11,31691
ARCLK	0,124077	**SAHOL**	0
ASYAB	0	**SISE**	0
BIMAS	10,93315	**SNGYO**	0,9399
DOHOL	0,683703	**TCELL**	24,39628
ENKAI	6,109964	**THYAO**	2,071397
EREGL	0	**TKFEN**	0
GARAN	0	**TOASO**	0
HALKB	0	**TTRAK**	4,897406
IHLAS	4,54465	**TUPRS**	7,498326
ISCTR	0	**VAKBN**	0
KCHOL	0	**YKBNK**	0
KRDMD	0		

We determine the composition of portfolio as in the Table 4.4 We give the expected return, variance and standard deviation of optimal portfolio in Table 4.4

Table 4.4. Results of Mean Variance Optimal Portfolio

Expected Return(%)	0,093
Variance	0,000244904
Standard Deviation	0,015649408

We can plot efficient frontier using the portfolios that provides expected return with minimum risk. We determine Mean Variance model in Excel Solver with the same constraints for different levels of expected return. After we find risk and expected returns we plot the efficient frontier. In Table 4.5 we give risk and expected returns of portfolios that place in efficient frontier.

Table 4.5. Risk and Expected Return Values of Some Efficient Portfolios

Portfolio	Expected Return(%)	Std Deviation
1	0,01	0,019430857
2	0,02	0,018226957
3	0,04	0,016838017
4	0,06	0,016151672
5	0,08	0,015749183
6	0,093	0,015649373
7	0,1	0,015645584
8	0,12	0,015841829
9	0,14	0,016435226
10	0,16	0,017685779
11	0,18	0,027966027
12	0,2	0,031254627

We determine the expected returns and standard deviations for 12 different efficient portfolios. If we put these values in risk-return axis we procure the efficient frontier.

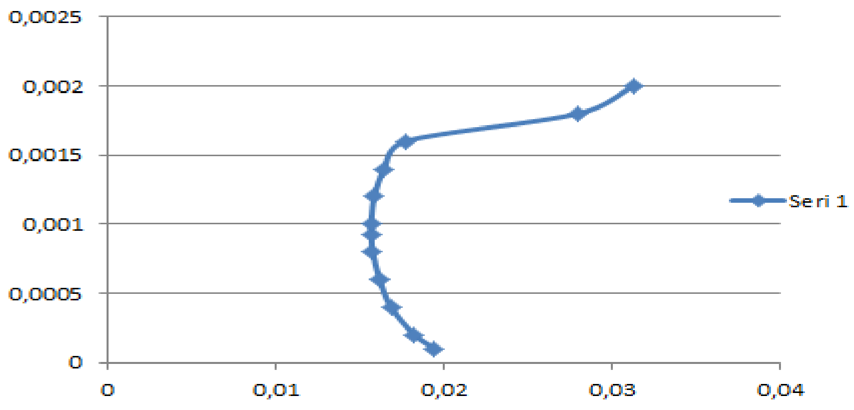

Figure 4.1. Efficient Frontier

5. CALCULATION OF OPTIMAL PORTFOLIOS' RISK with VaR APPROACH

5.1. Measuring VaR of Equal Weighted Portfolio

Determining risk of a portfolio helps investor to make quick decisions about portfolio. Investors want to know how much they loose rather than how much they get. VaR calculates the worst expected loss over a given horizon at a given confidence level under normal market conditions. It is one of the most widespread risk measurement model in use. Value at Risk gives a single number, summarizing all types of risks of a portfolio have. This provides great simplicity in use.

In this section we calculate VaR of the equally weighted portfolio using Historical Simulation method. Historical Simulation Method defines past performance of portfolio is a good indicator of near future. So we have to calculate past returns.

- First we calculate returns of assets indivudally and then we can find the total returns of portfolio using (3.5.2.1).
- After we calculate the returns we sort these values from lowest to highest.
- TheVaR at 95% confidence level is 5% lowest value in the series of simulated returns.

In Table 5.1 we see a sample of 1000 simulated data. The 5% lowest data which is 50th data in this order.

Table 5.1. A Sample for return of equally weighted portfolio(%)

Order	Portfolio Return	Order	Portfolio Return
1	-9,068931	481	0,1356663
2	-8,5288995	482	0,14412
3	-7,6466767	483	0,1525043
.	.	484	0,1529891
.	.	485	0,1569812
45	-3,186834	488	0,1617842
46	-3,1659244	.	.
47	-3,1073212	.	.
48	-3,085506	993	4,8103095
49	-3,059801	994	4,8744854
50	-3,0485475	995	4,8817647
.	.	996	5,1965891
.	.	997	7,4937212
.	.	998	7,665051
.	.	999	10,84314
.	.	1000	12,4812815

As we mentioned above 50th data of this sample is the VaR of the portfolio. The VaR of equal weighted portfolio at 95% confidence level is -3,05%. In Figure 5.1 we give histogram of return of portfolios and VaR of Portfolio.

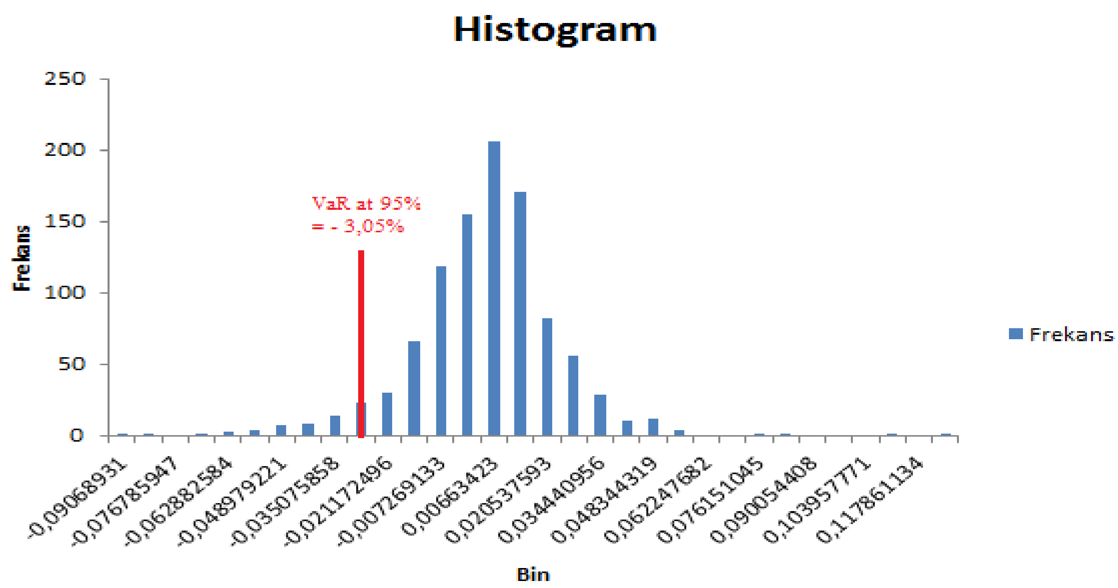

Figure 5.1 Histogram of Equally Weighted Portfolio Returns

5.2 Measuring VaR of Mean-Variance Optimal Portfolio

We determined an optimal portfolio using Mean Variance Method which is defined in section 4.4. We measure VaR of this portfolio to determine its expected loss. We follow same steps in section 5.1 to calculate VaR of Mean Variance Optimal Portfolio. In table 5.2 we give a sample of return of optimal portfolio.

Table 5.2. A Sample for Returns of Mean Variance Optimal Portfolio(%)

Order	Portfolio Return	Order	Portfolio Return
1	-6,658468446	481	0,086726231
2	-5,673486159	482	0,087415598
3	-5,499586728	483	0,090831877
.	.	484	0,093277394
.	.	485	0,095634001
45	-2,778165837	488	0,097390381
46	-2,692368059	.	.
47	-2,655731667	.	.
48	-2,63417772	993	4,11556372
49	-2,618363672	994	4,121141222
50	-2,601843461	995	4,326660393
.	.	996	4,490358469
.	.	997	5,156454202
.	.	998	6,081263156
.	.	999	8,89950126
.	.	1000	11,03884991

The 50th data is the VaR of the portfolio at 95% confidence level which is equal -2,602%. This means value of our optimal portfolio can reduce maximum 2,602%. In Figure 5.2 Histogram of optimal portfolio returns and VaR of the portfolio.

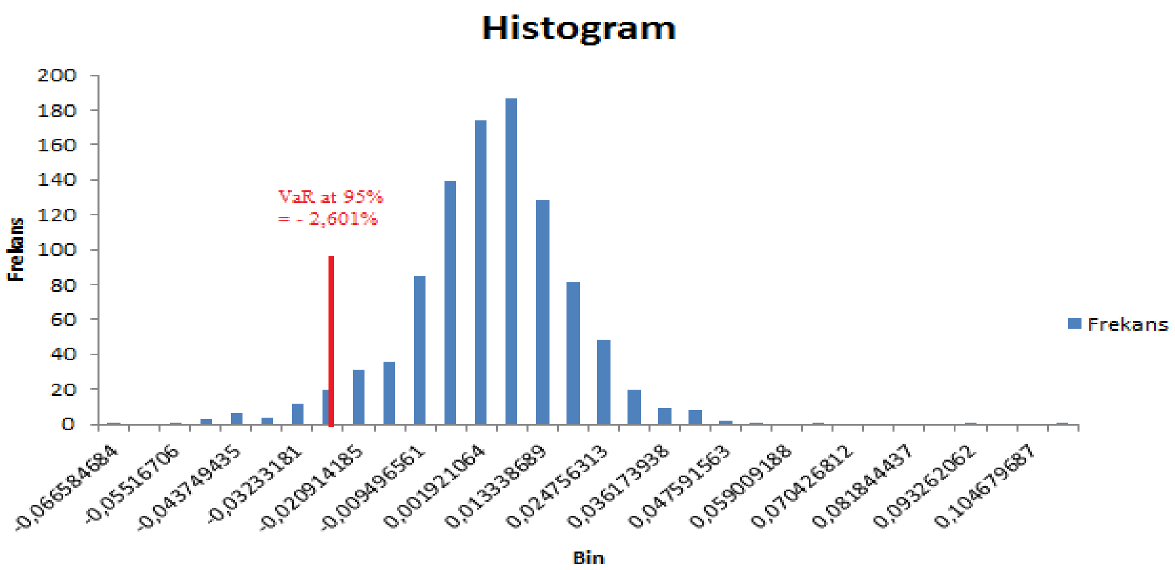

Figure 5.2 Histogram of Mean Variance Optimal Portfolio

6. CONCLUSION

Traditional Portfolio Theory presented best investment idea until 1950s. It supports diversification of asseets to reduce the risk of the portfolio. But this idea has some weaknesses. If we increase the number of the assets without analysing the correlations between them, we cause to reduce the expected return of the portfolio. Harry Markowitz published *"Portfolio Selection"* in 1952. In this article Markowitz presents a solution for reducing expected returns problem. He explains in his article that investors must consider correlation between assets. He also gives a mathematical framework for portfolio optimization. It is called Mean Variance Method which is the basis of Modern Portfolio Theory.

In this work first we composed a portfolio which contains all the 27 assets that are trading in ISE-30. We calculate the expected return of this equal weighted portfolio. Then we build up another portfolio using Mean Variance Method. We use some constraint while composing optimal portfolio with Mean Variance approach. We use the same expected return as equal weighted portfolio to compare performance of these two portfolios. We also block short selling with another constraint. To compare risk of these two portfolios we use Value at Risk as a risk measure. VaR calculates maximum expected loss of the portfolios over a time period. It gives a single number summarizing all types of risk that aportfolio can have.

After calculations VaR of two optimal portfolios we determine Mean Variance optimal portfolio has lower risk than equally weighted portfolio at the same expected return level. Equally weighted portfolio has 27 different assets in it however Mean Variance optimal portfolio has just 13 assets which are correlated with each other. This result shows increasing just the number of assets in portfolio doesn’t reduce the risk efficiently.

REFERENCES

[1] Cooray, B. A. (September, 2003). *Understanding Risk Management & Portfolio Diversification.* Retrieved March 13, 2012, from http://bizlk.com/Articles/Risk%20Management%20&%20Portfolio%20Diversificati on.pdf

[2] Campbell J. Y., 2002. Strategic Asset Allocation: Portfolio Choice for Long-Term Investors, *Invited address to the American Economic Association and American Finance Association,* Atlanta, Georgia, USA, January 4, 2002.

[3] Rasmussen, M. 2003. *Quantitative Portfolio Optimisation, Asset Allocation and Risk Management.* N.Y.: Palgrave Macmillan.

[4] Elton, E. J., Gruber, M. J., Brown, S. J., Goetzmann, W. N., 2003. *Modern Portfoilo Theory and Investment Analysis* (6th ed.). USA: John Wiley&Sons Inc.

[5] Elton, E. J., Gruber, M. J., 1997. Modern Portfolio Theory, 1950 to Date. *Journal of Banking & Finance, 21,* 1743-1759.

[6] Markowitz, H., 1952. Portfolio Selection. *The Journal of Finance*, 7*,* 77-91.

[7] Gelderman, C. J., 2003. *A portfolio approach to thedevelopment of differentiated purchasing strategies.* Maastricht: Universiteits Drukkerij/Technische.

[8] Kale, J. K., 2009. Portfolio Optimization Using The Quadratic Optimization System. *Managerial Finance, 35(5),* 439-450.

[9] Fabozzi F. J., Gupta, F., Markowitz H. M., 2002. The Legacy of Modern Portfolio Theory. *The Journal of Investing. Fall 2002,* 7-22.

[10] Koch, A. K., 2007." Mean Variance Analysis", Lecture 3 and Lecture 4, University of London, unpublished.

[11] Markowitz Mean-Variance Portfolio Theory. Retrieved March 27, 2012 from http://www.math.washington.edu/~burke/crs/408/fin-proj/mark1.pdf.

[12] Sevinç, E., 2007. İMKB-30 Endeksinde Yer Alan Menkul Kıymetlerden Ortalama-Varyans Modeline Göre Optimal Portföy Oluşturulması ve Riske Maruz Değer Yaklaşımıyla Portföy Riskinin Hesaplanması, *Master Thesis, İst*anbul University Institute of Social Studies, İstanbul.

[13] Mausser, H., Rosen, D., Efficient Risk/Return Frontiers for Credit Risk. *Algo Research Quarterl, 2,* 35-48.

[14] Value at Risk(VaR). Retrieved March 17, 2012, from http://pages.stern.nyu.edu/~adamodar/pdfiles/papers/VAR.pdf.

[15] Longerstaey, J., Spencer, M., 1996. *RiskMetrics—Technical Document* (4th ed.). USA.

[16] Dowd, K., 2000. *Accounting For Value at Risk In Bank Portfolios.*Notthingham,

[17] Dowd, K., 2002. *An Introduction to Market Risk Measurement.* West Sussex: John Wiley & Sons Ltd.

[18] Basle Committee on Banking Supervision, 1996. Amendent to the Capital Accord to Incorporate Market Risks.

[19] Berry, R., (n.d.). *Value-at-Risk: An Overview of Analytical VaR*. Retrieved April, 2, 2012 from http://www.jpmorgan.com/tss/General/email/1159360877242.

[20] Jorion, P., 2000. Value at Risk: *The New Benchmark for Managing Financial Risk.* USA: McGraw-Hill.

[21] Berry, R., (n.d.). *An Overview of Value-at-Risk: Part III – Monte Carlo Simulations VaR* Retrieved April 2, 2012 from http://www.jpmorgan.com/General/RiskManagement/11593806.

ŞANSAL BAŞARAN

Personal Information

Date of Birth	: 15.09.1989
Place of Birth	: Bornova
Nationality	: Turkish
E-mail	: sansalbasaran@gmail.com

Education Information

University(Undergraduate) 2007-2012	Istanbul Techincal University *Mathematics Engineering*
High School 2003-2007	Izmir Ataturk High School

Job Experience

08/2011-09/2011	DHMI Atatürk Airport *IT Intern*
07/2010-08/2012	Eureko Insurance Company *Finance Intern*
06/2010 –07/2010	Istanbul Techincal University Faculty of Science and Letters *Finance Intern*

Foreign Languages

English, German(Basic)

Computer Skills

Microsoft Office, C Programming Language, Java(Basic), PHP, MySQL, MS SQL(Basic)