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Comparison of VaR and CVaR Criterion on Risk Optimization

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FEN - EDEBİYAT FAKÜLTESİ

BİTİRME ÖDEVİ

Risk Optimizasyonu Konusunda VaR and CVaR Metotlarının Karşılaştırılması

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PREFACE

I would like to thank to my supervisor Dr. Eti Mizrahi for all her help and interest in my project.

Kübra DUMAN

SUMMARY

Risk management is a strategic issue for banks. Banks with proper risk management systems they control their risks and reduce their losses. In addition to these, since they have considered the risk factors in their profit analysis, their decisions and moves become more profitable. Therefore, especially in recent years, in order to determine the effects of the asset’s volatility the control of the market risk has an increasing importance. For this purpose in financial market lots of methods are used, but VaR and its alternative CVaR is more preferred than others.

Value at Risk (VaR) is a statistical method, that gives the expected maximum loss for a specific confidence level and time horizon. Since Value at Risk doesn’t obtain reliable results for non-normally distributed asset’s return, Artzner and Uryasev introduced CVaR as an alternative to VaR. CVaR is interested in the loss exceeding VaR and it calculates the maximum average loss for given confidence interval. So, CVaR values are greater than VaR values. Since the aim of this study is forming a handbook about VaR ve CVaR, we presented theoretical knowledge about VaR ve CVaR. For this purpose, we mentioned about their calculation methods, their imortance on risk management, how coherent risk metrics are they and comparisions of them. In this handbook in addition to theoretical information about VaR and CVaR, there is also an application about the calculations of these methods for better understanding. For this application we have used ISE 100 index data between the dates 04.01.1999 – 27.01.2000 and 03.01.2011 - 30.12.2011 and we have used MATLAB, Eviews and ECVaR programs for the calculations.

ÖZET

Risk yönetimi bankalar için stratejik bir konudur. Bankalar doğru risk yönetimi ile bir yandan risklerini kontrol edip kayıplarını azaltırken diğer yandan karlılık analizlerinde risk faktörlerini de inceledikleri için daha karlı hamleler yaparak hisselerini büyütürler. Dolayısıyla, özellikle son yıllarda finansal yatırımlardaki oynaklıkların etkilerini kontrol etmek amacıyla piyasa riskinin kontrolü daha da önem kazanmıştır. Bu amaçla piyasada kullanılan pek çok yöntem olmakla birlikte VaR ve ona alternatif olarak sunulan CVaR yöntemleri daha çok tercih edilmektedir.

Belirli bir güven aralığında muhtemel maksimum zarar miktarını ölçen istatistiki yönteme Riske Maruz Değer (VaR) denmektedir. VaR normal dağılıma sahip olmayan portföy getirileri için güvenilir sonuç vermediğinden Artzner ve Uryasev CVaR yönteminden bahsetmişlerdir. CVAR, VaR'ı aşan kayıplarla ilgilenir ve verilen güven aralığı için ortalama maksimum kaybı hesaplar. Dolayısıyla VaR değerinden daha büyük CVaR değeri elde edilir. Bu çalışmanın amacı VaR ve CVaR ile ilgili bir el kitabı oluşturmak olduğundan bu çalışmada VaR ve CVaR ile ilgili teorik bilgi sunduk. Bu amaçla VaR ve CVaR metodlarından, hesaplama yöntemlerinden, ne derece tutarlı risk metriği olduklarından ve risk yönetimindeki yerlerinden bahsettik ve bu yöntemlerin karşılaştırmasını yaptık. Verilen teorik bilginin yanında ayrıca verilen uygulama örneği ile bu teorik bilgi pekiştirilsin istedik. Bu uygulamada 04.01.1999 – 27.01.2000 ve 03.01.2011 – 30.12.2011 tarihleri arasındaki IMKB 100 fiyat endeksi verilerini MATLAB, Eviews ve ECVaR programlarında kullanarak çeşitli hesaplamalar yaptık ve grafikler elde ettik.

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1. INTRODUCTION

With the increasing financial fragility and the extensive use of derivative products in developed countries financial world, risk management in selecting the financial investment vehicles for minimizing risk has become a necessary issue with everyday increasing importance. Since risk is a measurable and manageable concept, risk is constantly analyzed by investors. So the need to determine and measure the risk of the financial assets with the help of lots of methods becomes a necessary issue [26]. In a specific time interval, the estimation of the holding asset's loss and profit values is important for finance market. Nowadays VaR, which is accepted and used widely by financial market and CVaR, as an alternative to VaR are the methods to determine and minimizing the risk of the portfolio.

VaR Models have an increasing use since it was first introduced by JP Morgan in 1994. VaR calculates for the financial institutes the expected maximum loss over a time specific time interval for a given confidence level. Because of the proposition of the Basel Committee's adoption of VaR, its simplicity in calculation and presentation its use have increased dramatically [3]. In addition to VaR, in this handbook we also have informed about CVaR, which is interested in losses exceeding VaR. It calculates the risk as the mean value of the percent worst confidence level. According to Artzner since VaR is not coherent, CVaR is suggested to use instead of VaR [7]. As Rockafeller and Uryasev also mentioned CVaR has better results on risk optimization because of the satisfaction of convexity rule [24, 25]. Although CVaR is better than VaR on satisfying the coherent risk metrics properties, VaR is preferred much on risk management because of its easy calculations and widely acceptance [2, 3, 26].

Since VaR is really popular in finance and risk management, there are lots of studies in more varieties than CVaR on the literature [1, 2, 3, 4, 7, 10, 12]. And as its alternative CVaR the studies are mostly after 2000s. Especially Artzner, Uryasev and Rockafeller have studied mostly about CVaR and given important mathematical properties about it [3, 24, 25, 26, 27, 28].

In this graduation project we aim to form a handbook about Value at Risk (VaR) and Conditional Value at Risk (CVaR) on risk optimization. With this Project we aim to provide a better understanding about VaR and CVaR with the theoretical information. In Section 2 we have addressed Risk Management by giving information about Technical Risk Ratios and Coherent Riskmetrics Concept. In Section 3 we have mentioned about what is VaR, its popularity, its parameters, its calculations, its measurement methods and other market methodologies using similar VaR techniques. In Section 4 we have given information about what is CVaR, its approaches, its characteristics and its calculations. In Section 5 we have compared the methods VaR and CVaR with positive and negative sides. In Section 6 there is an application for VaR and CVaR calculation and comparison where ISE 100 index data are used.

2. RISK MANAGEMENT

Especially for businesses, financial institutions and even for individuals the estimation of losses on the value of assets that may occur in certain time period has a great importance. Increased risk requires an effective risk management system [26]. Risk management is a technique of identification and elimination of possible risks and if the elimination is not possible, risk management tries to reduce the risk and its losses. Especially for banks risk management is a strategic issue. With the help of risk management banks control the loss and reduce their risk and moreover with the analysis they get more profitable products and add more value for their shareholders. Banks with strong risk management analyses in detail their market risk (market prices movements such as interest rate and currency rate cause ill equity and potential loss), credit risk (losses due to problematic credits), operational risk (in general all other risks such as errors, omissions, information technology systems corruption, legal reasons or new arrangements which affect the profitability in the sector), determine their loss for possible crisis beforehand and take precaution in time. These banks, measuring their risk developed analytical methodologies such as Value at Risk, Expected Loss, Sharpe Ratio, Alpha-Beta Parameter and RAROC. With these methodologies they monitor and report the risk with measurements and they control and manage it with crisis scenarios. In big banks risk is evaluated with these methodologies and it is used on strategic decision making process and daily transactions. Value at Risk and Conditional Value at Risk are just two of them. Especially in banking business VaR is one of the basic risk measurement techniques [6]. There are lots of ways to estimate the market risk, whose points are determined in the following table [8].

CASE 1	CASE 2	DECISION
Can user accept the assumption of normality; is it reasonable to assume that market movements follow the normal distribution?	Does the value of positions change linearly with changes in market prices?	
YES	YES	Standard measures of risk such as duration and convexity can be used comfortably.
NO	NO	Scenario analysis combined with simulation techniques must be used.
YES	NO	Combination of statistical tools and simulation techniques should be preferred.

Table 2.1: Decision on Market Risk Estimation

2.1 Technical Risk Ratios

Since there is a relation between risk and reward, investors uses technical risk ratios to assess the risk adjusted return on an investment. These are all statistical measurements used in modern portfolio theory (MPT). All of these indicators are intended to help investors determine the risk-reward profile of a mutual fund.

Definition 2.1.1: R- Squared

It measures the correlation degree between the movements of the assets in relation to the index. R-Squared values changes from 0 to 100. An R-squared of 100 means that all movements of a security are completely explained by index's movements. High R-Squared value makes beta more useful and lower R-Squared value insensible [16].

Definition 2.1.2: Beta

Beta is a measure of the volatility or systematic risk of a security or a portfolio in comparison to the market as a whole. Beta is used in the capital asset pricing model (CAPM), a model that calculates the expected return of an asset [6]. A beta of 1 indicates that the price movement of the stock will follow the markets. In other words, if the index rises by 5%, then the security with a beta of 1 will also show a similar increase. A beta of less than 1 means that the stock is less volatile than the market, while a security with a beta greater 1 will show higher volatility than the market [15].

Definition 2.1.3: Alpha

Alpha is a coefficient which measures risk-adjusted performance and compares it to a benchmark index by considering the risk due to a specific security rather than the overall market. A positive alpha of 1.0 means the fund has outperformed its benchmark index by 1%. Correspondingly, a similar negative alpha would indicate an underperformance of 1%. For example, if a CAPM analysis estimates that a portfolio should earn 10% based on the risk of the portfolio but the portfolio actually earns 15%, the portfolio's alpha would be 5%. This 5% is the excess return over what was predicted in the CAPM model [14].

Definition 2.1.4: Standard Deviation

This metric measures the fluctuation of a dataset from the mean. The more variant the dataset, the higher the deviation from the mean. It is an important tool for the computation of Sharpe Ratio. It measures an investment's variability of returns; that is, its volatility in relation to its average returns. It is a simply measure of volatility and as a measure of the probability of loss is of limited use [8].

Definition 2.1.5: Sharpe Ratio

The Sharpe ratio is developed by Nobel laureate William F. Sharpe to measure risk-adjusted performance. The Sharpe ratio is calculated by subtracting the risk-free rate from the rate of return for a portfolio and dividing the result by the standard deviation of the portfolio returns. The idea of the ratio is to see how much additional return you are receiving for the additional volatility of holding the risky asset over a risk-free asset. The greater a portfolio's Sharpe ratio, the better its risk-adjusted performance has been [8]. The Sharpe ratio formula is:

$$\mathrm{S.R} = \frac{\mathrm{Rm} - \mathrm{Rf}}{\mathrm{Vm}}$$

Rm : rate of return of investment m

Rf : risk-free rate of return

Vm : Standard deviation of instrument m

It is a reward-risk ratio. It measures the extent to which the return of an investment (above risk-free return) exceeds its volatility. For an investor it is more useful as a relative measure, in comparing the ratio of one investment to that of another. This measurement is very useful because although one portfolio or fund can reap higher returns than its peers, it is only a good investment if those higher returns do not come with too much additional risk. A negative Sharpe ratio indicates that a risk-less asset would perform better than the security being analyzed [8, 17].

2.2 Coherent Risk Metrics

A measure is an official action that is done in order to achieve a particular number. A metric is our interpretation of the achieved number. Risk metric is basically a measure of uncertainty in a distribution, so it doesn't say anything about risk attitude of an investor, because it is not a premium risk. Duration, delta, gamma, convexity, beta and volatility are examples for metric and the operations with which we quantify them are measures. Risk measures tend to be categorized according to the risk metrics that they support. Artzner and his coworkers introduced that a coherent risk metric should satisfy the following axioms [2]. In the following axioms we use the symbol ς to symbolize arbitrary risk metric and we use A and B as two financial assets.

Property 2.2.1: Monotonicity

It means that mean and deviation of the series don't change with the change of time. If an investment A is dominates another investment B in the sense that the probability of the return is never greater with investment B than with investment A [28].

$$\varsigma\ (A) \leq \varsigma\ (B) \text{ when } A \leq B$$

Property 2.2.2: Sub-additivity

For the sub-additivity rule, the diversified portfolio's risk must be less than the corresponding weighted average of the risks of the constituents [2].

$$\varsigma\ (A + B) \leq \varsigma\ (A) + \varsigma\ (B)$$

Property 2.2.3: Homogeneity

Some authority believes that double investments will also double the risk [2]. So for any positive constant κ the homogeneity axiom necessitates,

$$\varsigma\ (\kappa A) = \kappa\, \varsigma\ (A) \text{ for } \kappa > 0$$

Property 2.2.4: Risk-free Condition

In the risk-free condition we guarantee that we will not lose any money. So, if we have both risky and risk-free security together, according to risk-free axiom our net capital at risk should be the addition of our risky security's capital at risk and our risk-free security's capital [2]. More generally, let divide our capital into an investment A and a risk-free return Ψ. Then we can calculate the net capital as,

$$\varsigma\,(A+\boldsymbol{\Psi}) = \varsigma\,(A) - \boldsymbol{\Psi}$$

In addition to these coherent risk metric properties, there is also another important property for VaR and CVaR calculations, which is convexity rule. The key point on convexity rule in optimization is that the guarantee for local minimum point is also global minimum point for the return function [24, 28].

$$\varsigma\,((1\text{-}\lambda)\,A + \lambda\,B) \leq (1\text{-}\lambda)\,\varsigma\,(A) + \lambda\,\varsigma\,(B) \text{ for any } \lambda \in\,]0,1[$$

3. VALUE AT RISK (VaR)

Statistically VaR is defined as portfolio's profit and loss distribution over a sample point, which varies over time as market conditions and the portfolio's composition [7]. VaR is an estimate of an amount of money, which depends on the probabilities. So there is no certainty. VaR calculates the volatility of an investor's asset. So, the greater the volatility, the greater VaR figure, the higher probability of loss [8].

Definition 3.1: VaR

VaR is a procedure designed to forecast the maximum expected loss for an asset or a portfolio under certain assumptions over a target horizon and given a statistical confidence limit.

So, if the selected confidence level is α in a given time period for the profit-loss distribution, The VaR is corresponds to the 1- α lower-tail level. Basically VaR answers the question: "Under normal market conditions how much money will be lost for the holding portfolio in a certain time with an x% probability?". For example, for an investor with an amount of 100 million TL, if we assume the confidence interval as 99%, the time period as 1 day and the VaR value is calculated as 1 million TL. Under normal market conditions this investor will lose maximum 1 million TL with a probability of 99%. Stated in other words his maximum loss will exceed 1 million TL with a probability of 1% [12].

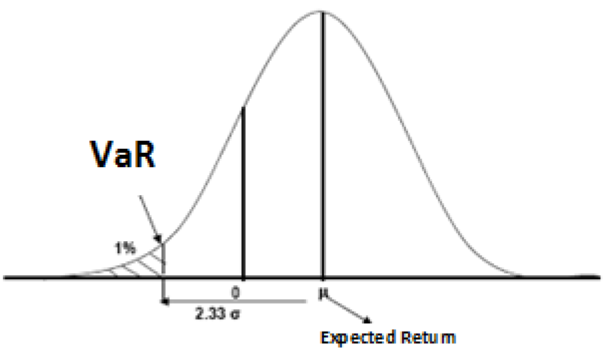

Figure 3.1: Value at Risk

3.1 Why VaR is so Popular?

Scientific studies show that with the correct VaR measurements many of businesses can have been preserve themselves from financial risks previously. Risk measurement is important for the continuity of the businesses. Therefore, the state, large scale trading operations, retail banks, insurance companies, institutional companies and non-financial enterprises such as independent auditors, the trade unions are interested in VaR figures. Because of the interest on VaR, it has subsequently become one of the most important and widely used measures of risk. It is preferred to use due to conditions below [3, 26].

1. Simplicity of the VaR calculation is appealing.
2. It is accepted by the top management.
3. Its use is encouraged by BIS (Bank for International Settlement).
4. It can show the total portfolio's risk only with a number, which make easy to understand and comment for the managers.
5. At a macro level it can be used also for portfolios like single investments at a micro level.
6. VaR estimates the total effect for market risk, such as interest rate, currency rate, share price, inflation.
7. VaR technique is not only a risk management tool, it is also important on reporting the business's information related with their risk, performance analysis and outcome's adaptation of risk.

3.2 VaR as a Coherent Metric

Many risk metrics, which are expressed in relative terms such as volatility, are not coherent, because they don't satisfy risk-free condition. VaR is coherent risk metric under special assumptions about the distribution of returns. If the returns are normally distributed then VaR is a coherent metric, but more generally VaR is not coherent because of not satisfying sub-additivity rule [2].

Although VaR is very popular on use, it also doesn't satisfy the convexity condition. So it is not easy to measure an optimal point for a return function, which has more than one local minimum point.

3.3 How to Calculate VaR?

There are different methods to calculate the VaR, which are the parametric method, historical simulation, and Monte-Carlo simulation method. In all the three methods the confidence interval is accepted as 95% or 99% [1]. Variance-Covariance Method as the parametrical approach is the simplest and most widely used method of VaR. As we mentioned before the basic assumption of this method is the normal distribution of the returns and their linear effect on the portfolio's value [4, 26]. Therefore according to kurtosis and skewness values regarding as asset's returns normal distribution's characteristics are searched. In this method, the parameters are the portfolio value (P), the standard deviation of the return's distribution (volatility of the risk factors) (σ), holding period (t) and selected confidence interval (α). For a single asset the VaR is calculated with the following formula [1].

$$VaR = P * \sigma * \sqrt{t} * \alpha$$

P : Value of portfolio

t : Holding period

σ : Volatility of the risk factors

α : Confidence interval

Let calculate the VaR figure for an investment of 10,000 TL with a daily volatility of 0.02 and confidence level of 95% on 100 days holding period.

VaR = 10.000 x 0.02 x 10 x 1.645, which is equal to 1,040.4 TL. Here the figure 1.645 is the z statistics value equals to 95% confidence interval on normal distribution.

As a result, 10,000 TL on 100 day time period with 95% probability the maximum expected loss will be 1,040.4 TL [12].

With the increase of assets variety on the portfolio, because of the correlations, which must be considered, the VaR figure will be difference [1, 11]. Portfolios with more than one financial asset VaR is calculated with matrix multiplications.

$$\sigma = x * C * x^T$$

x : The weight of the asset in the portfolio

C : Variance – Covariance matrix

Variance – Covariance matrix (C) is calculated with the following formula, which is the multiplication of correlation matrix of the assets and standard deviations of assets.

$$C = \begin{bmatrix} \sigma 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma n \end{bmatrix} \begin{bmatrix} 1 & \cdots & \rho 1n \\ \vdots & \ddots & \vdots \\ \rho n1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \sigma 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma n \end{bmatrix}$$

After the calculation of the volatility of the portfolio VaR is calculated with the below formula.

$$VaR = P * \sqrt{x * C * x^T} * \sqrt{t} * \alpha$$

Instead of using the weights of the assets if the asset’s value is used, then portfolio value doesn’t take place in the formula. In this condition the formula transforms as the following.

$$VaR = \sqrt{P * C * P^T} * \sqrt{t} * \alpha$$

Additionally, VaR can be calculated as using VaRs of each asset with their correlation matrix.

V : VaRs of assets

$$VaR = \sqrt{V * \rho * V^T}$$

$$V = \begin{matrix} V1 \\ \vdots \\ Vn \end{matrix} = \begin{bmatrix} P1 & * \sigma & * \alpha \\ \vdots & \vdots & \vdots \\ Pn & * \sigma & * \alpha \end{bmatrix}$$

$$VaR = \sqrt{\begin{bmatrix} V1 \\ \vdots \\ Vn \end{bmatrix} * \begin{bmatrix} 1 & \cdots & \rho 1n \\ \vdots & \ddots & \vdots \\ \rho n1 & \cdots & 1 \end{bmatrix} * \begin{bmatrix} V1 \\ \vdots \\ Vn \end{bmatrix}^T}$$

3.4 Parameters used in VaR Measurement

There are parameters used in VaR measurement, which should be determined before the measurement. These parameters are Holding Period, Sampling Period, Confidence Interval, Symmetry to Normal Distribution, which are relative and defining of these parameters depends on the investor's situation and characteristics of holding portfolio [12, 26].

3.4.1 Holding Period

According to BDDK, banks have to determine the holding period as 10 days. Determination of the holding period depends on the characteristics of the portfolio. If the portfolio's value changes rapidly or it is affected from price differences, with the greater holding period creates lower VaR figures [26]. Value at Risk is calculated for a particular holding period. There is a direct proportion between the holding period and market risk. With the longer holding period the volatility will be higher. There are three main factors that influence the determination of holding period. The first one is market liquidity. Position should be taken short if the market can buy the investor's asset easily in liquid markets, and if it is not possible long position should be taken. The other two factors, which affect the holding period, are the normality assumption on the model and variation frequency in the portfolio's content requires determining lower holding period. Although the normality assumption doesn't satisfy, acceptance of normality assumption can be valid only in short holding period. If in long term the probability of the frequency vibration in the portfolio's content is higher, than it requires also shorter holding period. In Turkey, because of the lack of liquidity in the markets, to measure the risk more reliable longer holding period is more preferable. Holding period in VaR measurement is reflected as to the square root of time. This relation depends on the approach 'Geometric Brownian Motion' [12].

3.4.2 Sampling Period

The success of VaR calculation depends on the data set, whose volatility and correlations are used in VaR calculation. For different data set the calculated volatility will be different and owing to different volatility also the VaR will be different. Basle Committee has identified the sampling period as minimum 252 working days, one year, and obliged to update the data set periodically and according to Basle Committee VaR measurements should be done with newer data sets [12, 26].

3.4.3 Confidence Interval

Value at Risk figure is calculated under the assumption of normal distribution of asset's return. Under this assumption confidence interval gives the probability of maximum portfolio's loss. VaR is interested in only with the loss, so the left tail of the normal distribution should be considered on the calculations. According to application demands confidence interval can be determined between 90% and 99%. Firms prefer to choose lower confidence interval for the system validity and higher confidence interval for the risk management and capital adequacy [12]. Like Basel Committee BDDK also wants the VaR calculations with a confidence interval of 99% and one tail of distribution [26]. According to report published BDDK in 10.02.2001, in Turkey VaR must be calculated with %99 confidence interval, minimum 10 days holding period and 252 weekday's data (1 year). JP Morgan prefers to use 95% confidence interval in its calculations. With the higher confidence interval banks obtain greater VaR figures [1, 29].

3.4.4 Symmetry to Normal Distribution

The most important assumption on the VaR calculations is the assumption of normality, normal distribution of asset's return. But in practice, financial gain doesn't fit normal distribution. The biggest problem here is that distribution shows fattailed, skewness and kurtosis features. In this case, normality assumption remains insufficient to determine the peak and the tail area of the distribution. Fattailed shows that the frequency of extreme values is high. In other cases, if the distribution doesn't show symmetry with the normal distribution, then the normality assumption is no more valid. These issues are tested with statistical tests [26].

Basically normal distribution depends on two parameters. One of these parameters is the mean of the distribution (μ) and the other one is the standard deviation of the distribution (σ). In every shape the area under the normality curve, which is broad or narrow doesn't important, equals to 1 or 100%. The area to the right of the mean shows it will be more than the mean with 50% and the area to the left of the mean shows it will be less than the mean with 50%. Skewness and kurtosis of distributions are calculated with the following formulas [12].

Skewness: $\tau = \frac{\mathrm{E}(x-\mu)3}{\sigma 3}$

Kurtosis: $K = \mathrm{E}(x - \mu)4 * \sigma 4 - 3$

These concepts are important on VaR and CVaR calculations for risk management and portfolio optimization. Kurtosis is a statistical measurement and it is used to describe the distribution of observed data around the mean. It describes the trends in the graphics, a high kurtosis portrays a graph with a fat tail and a low kurtosis portrays a graph with a skinny tail. Skewness describes the lack of symmetry corresponding to the normal distribution. There are two type of skewness, depending on whether the data points are skewed to the left or to the right, positive skewness and negative skewness. With the knowledge of skewness type future movement of the asset can be better estimated.

3.5 VaR Measurement Methods

The methods for VaR measurement can be classified as parametric and non-parametric methods. As Variance-Covariance method is called parametric method, Historical Simulation and Monte Carlo Simulation models are called non-parametric methods. In VaR measurement there are no strict lines in selecting method, which gives the best results. So, analysists preferred VaR methods through their needs [26]. Each method has its own advantages and disadvantages. Suitability of these methods depends on the structure of the return distribution and linear movements of returns. The situation with price changes can be defined with volatilities and correlation. And with the selected probability expected values on return can be obtained. In case, market price parallel to normal distribution, parametric method should be used. In cases where the normal distribution doesn't conform, calculation of expected value is really hard; because each expected value has its own probability and price movements has different return distributions. For these situations methods with simulation techniques gives better results. To generalize, portfolios with linear returns parametric method is used and portfolios with options or derivative products methods with simulation techniques are used.

In parametric method the calculations are made with assumptions for portfolios exposed to market risk, like not including option products, normally distributed asset's return and linearity between asset's returns. Historical simulation makes predictive models with the interest in price movements in the past. Based on simulation logic Monte Carlo method is a repeated random simulation of financial variable's price movements in a widest possible range of probability [12].

The weakest aspect of VaR calculation methods is that it doesn't show the worst situation. As you know, probability distributions represent possibilities within the selected confidence interval. But in the real world, although the probability is really weak, there is still a probability for events taking place outside of this area. It can't be never said that this events with very low probability will never occur. Another important issue is VaR doesn't show the total loss. It only gives us the maximum loss [26].

3.5.1 Variance – Covariance Method

This method is referred to variance-covariance method, which is the simplest and most widely used VaR method. The basic assumption of this method is normally distributed financial asset's returns have linear effects on the portfolio [4]. Therefore kurtosis and skewness values regarding as asset's returns are searched for the normal distribution [1]. It is assumed that in the parametric method there is a specific distribution. So financial institutions often apply this method and assume that there is a normal distribution for the returns. Based on the normality assumption, VaR value of the portfolio return can be calculated as the form of linear function of assets return's standard deviation [26]. This method's advantage is its simplicity on use. Parametric method involves basic matrix multiplication and even with a multiple number of assets in the portfolio it has rapid computability. At the same time it is criticized because of the assumption of normally distributed returns, which means variances and covariances doesn't change [26].

In this method, variance-covariance matrix, which is consist of basic parameters such as variance and covariance is formed by using past return series of the portfolio and in the future portfolio's expected loss is obtained with the calculation of VaR of risk factors.

Parametric method has two different types, Delta-Normal Method and Delta-Gamma Method.

3.5.1.1 Delta-Normal Method

This method is one of the often used methods on the parametric VaR calculations. In this method, it is assumed that portfolio's risk factors are distributed normally and portfolio's total risk is proportional with the combination of these risk factors. Basically, this method is used by RiskMetrics. We use also this method on our calculations.

3.5.1.2 Delta-Gamma Method

This method can be applied to large-scale portfolio with the linearity of the returns under the assumption of normality. But it doesn't give successful results on portfolios with options or structured products, which have gamma (second order sensitivity on the change of portfolio's unit value) and convexity. So this method addresses these deficiencies and takes into consideration the second order sensitivity of the portfolio and accepts quadratic assumption. On the other hand this method is not practical, because the required data increases geometrically [12].

3.5.2 Historical Simulation

Conceptually, historical simulation is the simplest approach to VaR calculation. This method doesn't make any assumption about the distribution of the assets and doesn't need parameters like variance or covariance. So, in this context it is a non-parametric method and it can be applied to all portfolios, linear or non-linear. In this method, scenarios are produced due to historical data [12]. The basic thought of this method is past events will see the light of day again in the future. It doesn't assume any distribution for the returns, so return's characteristics such as fat-tailed are taken into consideration in the original distribution. It is criticized because it assumes that past events will occur in the future again and it depends only on the selected past term for the estimations [1]. In this approach, the potential profit-loss distribution of the portfolio is obtained with the application of market factors' changes that have occurred during past N period. Historical simulation method takes a portfolio consisting of various assets on a particular time and evaluate several times. In this period, it uses the historical prices of the assets in the portfolio [26]. For example, with the 1000 daily data and with %95 confidence interval, it is expected that potential loss will exceed the VaR on 50 days or in other words on the %5 of the total day number. So, here VaR is the 51th biggest loss [12].

ADVANTAGES	**DISADVANTAGES**
It can be applied easily to non-linear positions.	This method requires intensive calculation processing.
It doesn't assume anything about distributions.	Random scenarios can lead the analyst to wrong interpretations.
	It considers only the past events; it doesn't take into consideration of possible changes in the future.

Table 3.1: Advantages and Disadvantages of Historical Simulation Method

The process of historical method calculation:

Step 1: Main risk factors on the portfolio are determined and asset's VaR are calculated with the market price.

Step 2: Historical data are provided for the risk factors occurred during the N-term (calculation period).

Step 3: VaR is calculated for the whole historical calculation period. All obtained hypothetical values are compared with the present portfolio value and the difference is gained.

Step 4: Obtained daily difference (profit/loss) is arranged in order from the worst to the best.

Step 5: The loss is determined corresponding to selected confidence interval [26].

3.5.3 Monte Carlo Simulation

Monte Carlo method based on the assumption that portfolio pricing process follows a particular model. This method is known as the most comprehensive and most powerful VaR methodology, because here the VaR figure contains both the non-linear relations in the portfolio and the impacts of any possible future changes. Additionally there is no constraint on the distribution for the returns [26].

This simulation can also be used like historical simulation for the portfolios with options, which doesn't have linear returns. But there is a key difference between these two methods. In historical simulation real past data is used for price changes on the hypothetical profit-loss process. But in the Monte Carlo simulation a statistical distribution is selected for the representation of possible price changes, and in this process random data is used [12].

Although the use and the calculation of Monte Carlo method are very laborious, it is more efficient than other methods because of its reliability on results. The main feature of the method is random numbers generation and re-appreciation of these numbers with the market prices. For the generation of random numbers, it is important to determine a distribution for the representation of changes of market factors. In this respect, although there is no necessary on using normal distribution, normal distribution is usually used in terms of its efficiency and consistency. The main disadvantages of this method are its complexity and time consuming characteristic. To have reliable results this method requires at least 10.000 random numbers for only one financial asset [1].

The VaR is calculated with the Monte Carlo method in nine steps:

Step 1: The portfolio is determined.

Step 2: The changes on returns of the portfolio's risk factors are calculated.

Step 3: The covariance matrixes of risk factors are calculated.

Step 4: Random numbers are created.

Step 5: In the covariance matrix Cholesky & Singular Value Decomposition matrix is created.

Step 6: With the multiplication of Cholesky & Singular Value Decomposition matrix's transposition and random price series, which are parallel to determined distribution, the relation of risk factors in the past are reflected to the new created price series.

Step 7: These price series are applied to the portfolio.

Step 8: After the evaluation of market prices founding hypothetical portfolio's return are arranged in order from the maximum loss to maximum profit.

Step 9: The loss is determined corresponding to the confidence interval [26].

3.5.4 Comparisons of VaR Measurement Methods

VaR is a useful method in portfolio's risk management and risk measurement systems, as long as it is understood and calculated correctly. To get the correct results in risk measurement, choosing the best method for the calculation of market risk is very important. There is no definite answer about which method should be used in VaR calculations. However, as we mentioned before the portfolio structure, ease of application, reporting techniques and reliability of the results are the ways to evaluate and select the most appropriate method for VaR calculation. If there is no derivative product in the portfolio then parametric method should be more suitable to use. But if there is a portfolio containing derivative products, then simulation techniques like Monte Carlo or Historical methods should be better to use to get more reliable results [12]. Although parametric VaR is stronger than other models in calculation speed and detection of risk, it is weaker in portfolios with non-linear instruments. Historical Simulation and Monte Carlo Simulation models give good results in VaR calculations on linear instruments. These methods are better than parametric method on modeling of high fluctuations. Historical Simulation is better than other models in non-normal distributions modeling.

Properties	**Variance-Covariance Method**	**Historical Simulation Method**	**Monte-Carlo Simulation Method**
Easy Computation	High	High	Low
Easy Application	High	High	Low
Report ability	Low	High	Low
Application (also) on derivative products	Low	High	High
Consideration of unexpected cases	Low	Low	High
Constraints	• Based on completely the normal distribution assumption • Not cover unusual market movements • Not good dealing with derivative products	• Historical data procuring difficulty • If the data set doesn't contain unusual price movements, then simulation can't contain unusual cases	• Higher modeling risk • Complex calculations and difficulty in understanding
Advantages	• Higher success in portfolio's with linear return	• Conceptually simple and comprehensive • Can be applied any kind of position	• Success in dealing with complex and non-linear positions

Table 3.2: Comparisons of VaR Methodologies

3.5.5 Additional Methods to Support VaR Measurement for Portfolio Optimization

3.5.5.1 Backtesting

Backtesting should be made to check the accuracy and measure the performance of the VaR calculations. In this method, the actual VaR figure is compared with the calculated VaR figure. Theoretically calculated VaR figure must be greater than the VaR figure obtained by backtesting procedure. If it doesn't satisfy, the trust on VaR reduces and it is called deviation. The effectualness of VaR methods can be tested statistically with the number of differences between the actual and expected deviations. The obtained z test statistic's value is compared with the value in the z table for the selected confidence interval. If the obtained z test value is greater than the value in the table than the model is not valid. There are two types of error for the backtesting process in statistics.

1. Error: Although the model is true, it is accepted as wrong, called α-error.
2. Error: Although the model is wrong, it is accepted as true, called β-error [12].

Deviation Number	
1-2-3-4	Green Zone
5-6-7-8-9	Yellow Zone
10+	Red Zone

Table 3.3: Classification of Deviation Numbers in Backtesting Procedure

Because of these anomalies, Basle Committee recommends the classification of these deviation numbers in green, yellow and red zone. If the number of deviations is equal or less than 4, these numbers fall to green zone and it means the model is reliable. If the number of deviations is between 5 and 9, these numbers fall to yellow zone and it means model should be reviewed. If there are more than 10 deviations, these numbers fall to red zone and it means the model is insufficient [6].

3.5.5.2 Stresstesting

Stress testing can be defined as a process, that unusual situations for losses can be identified and managed. It should be considered as a complement of other VaR methods [18]. Although under normal market conditions VaR is successful on estimation of maximum loss in the selected confidence interval, it may be insufficient in extraordinary market fluctuations. VaR assumes that market doesn't show any unusual movement, so it should be support with stress testing, which takes into account the changes of interest rate, exchange rate or stock prices in unusual periods like crisis terms or with unusual price change generation positions are evaluated. In this way, in the extreme market conditions the portfolios profit-loss situation can be analyzed. For example, the conditions for the portfolios profit or loss estimation on stress testing are extreme such as local currency depreciation with 40% or stock shares rapid decline of 30% in a short time. Stress models used in the process of stress testing can be said as follows:

1. Early Pushover Analysis
2. Optimization of Maximum Loss
3. Possible Scenario Model
4. Conditional Scenario Model
5. Historical Scenario Model
6. The Worst Case Scenario Model

Like all other methods this method has also advantages and disadvantages. As an advantage of this method, it may cover situations completely absent from the historical data. And it is a good method of calculation the worst-case effect of large movements in key variables. However, it is not as objective as the other methods. Bad or implausible scenarios can lead to wrong calculations of VaR. And it is no good applying it to large .and complex portfolios, because it handles correlation very weak [18].

3.5.5.3 Marginal VaR

Like Incremental VaR, Marginal VaR is also a special analysis that concentrates on where is the risk is higher for the portfolio. So, unwanted or non-profit positions or assets are decreased in the total portfolio with these analyses. It represents the change at VaR, if diversification or weights of the assets in the portfolio are changed. It has become one of the Standard tools used to eliminate the management risk and increase the returns [26]. This method is more important than other three main methods in terms of determining risk levels of the assets in the portfolio and comparing levels. This method measures the change on all fund's risk in the total risk, if it's weight increase 1% in the whole portfolio [1, 11].

From this point of view calculated VaR is important on forming efficient portfolios by decreasing risk [1]. So, it is calculated in 7 steps.

Step 1: The portfolio is determined.

Step 2: The return's changes of the portfolio's risk factors are calculated.

Step 3: Risk-free interest rate is determined.

Step 4: Risk-free return rates are calculated for the portfolio.

Step 5: Covariance matrix is calculated according to risk-free return rates.

Step 6: Weight vector is calculated for the portfolio.

Step 7: With the weight vector and selected confidence interval marginal VaR is calculated for the portfolio [12].

While VaR results are generally positive, Marginal VaR can be positive or negative. Positive Marginal VaR implies a reduction in total risk if a position is removed (or frozen). Negative Marginal VaR means that the total risk would actually increase by removal of the position (or freezing of the currency). In well-diversified portfolios many instruments may hedge each other and removal of a hedged position may actually result in a larger total risk. In fact, market neutral Portfolios (no net exposure) should show that almost all Marginal VaRs are negative since every position is hedging another. A Positive Marginal VaR might indicate a position that was not fully hedged. It is in these situations that Marginal VaR can be of greatest use by revealing hidden hedges and diversification [20].

3.5.5.4 Incremental VaR

As another point of view on VaR calculation we try to find which asset or which combination of them presents the maximum risk. So, users can change their positions and weights to modify and have more efficient VaR figures for their portfolio's risk optimization. But individual VaR is not enough by alone for this purpose [18]. Thus, there is another concept, Incremental VaR, which is used to identify the risk contributions of each position compared to the overall risk of the portfolio. IVaR helps to understand the dynamic between the various positions within a portfolio. The main drivers of VaR are the volatility of each position and the correlations between these positions. As we remove one position, the Variance-Covariance matrix will change accordingly and so will the VaR of the portfolio. Calculating IVaR can help grasp the dynamic of the correlations amongst all positions that compose a portfolio. As we remove a position and calculate its IVaR, we will be able to assess the significance of the interaction of that position with the other assets in the portfolio. The limitation of IVaR is that it cannot isolate the specific assets in the portfolio that are mainly affected by the removal of that position. There are two ways for existence of IVaR. If IVaR is positive, then this incremental position will add on more risk to the portfolio. If IVaR is negative, this incremental position will act as a hedge and be a factor of risk diversification for the portfolio. Along with VaR, IVaR has become a standard tool to make informed decisions about adding a new instrument to a current portfolio, and is widely used in Risk Management as well as in Investment/Portfolio Management. IVaR is mainly used to assess the (incremental) instruments that will provide the most optimal hedge to a portfolio [5].

3.6 Other Market Methodologies

Following to JP Morgan and its RiskMetrics other banks introduced their own VaR models into the market.

3.6.1 RiskMetrics

The best known system to measure and aggregate the risk is RiskMetrics system, which is developed by JP Morgan with the wish of showing risk and potential losses over the next 24 hours on daily one-page report. This system depends on standard portfolio theory, so it uses correlations estimating standard deviations and various correlations between the assets [18]. JP Morgan offered RiskMetrics as a free service in 1994 and after this date VaR is promoted as a risk management tool. The free availability encouraged smaller software providers to use RiskMetrics and make their work compatible with it. As a result, VaR becomes very popular on financial risk measurement and widely used [9]. JP Morgan would publish a methodology, distribute the necessary covariance matrix and encourage software vendors to develop compatible software. RiskMetrics is used 95% confidence interval for the VaR calculation.

3.6.2 CHARISMA

Charisma Model was developed by Chase Risk Management Analyser, Manhattan in 1996. Charisma forms distributions according to possible future prices change using historical data. This model identifies specific risks such as exchange and interest rate volatilities with price changes for last 100 days in these markets. Then the portfolio is revalued for each price change as they occurred from today’s price level, so 100 possible changes on portfolio’s value is created and then risk manager can determine the VaR figure for a selected confidence interval [8].

3.6.3 PrimeRisk

PrimeRisk was introduced by Credit Suisse First Boston in 1996, which is a model to forecast the volatilities and correlations. This model creates risk factors like volatility or prices for the calculation. PrimeClear, a front-end interface calculates and shows the VaR. And PrimeRisk also weights the data; it gives lower weightings to recent and distant days and higher weightings to intermediate day’s data, this is known as fractional exponential weighting [8].

3.6.4 RiskTürk

Risk Yazılım Teknolojileri is a leader firm on financial software and consulting. Market and credit risk management and asset liability management solutions are applied to lots of important Turkish bank since 2001. All of their software packages are customized according to customer's need. They provide risk management consulting and training, data integration and licenses on their projects. Their financial software programs are used by financial institutions in many different scales. Additionally, in recent times they have expanded their focusing on portfolio management, retirement fund and real sector [21]. For the market risk and asset liability management they provide solutions to AKBANK, Sekerbank, Is Yatırım, YapıKredi, TEB, Turkcell, and JP Morgan. For the credit risk they provide solutions to AKBANK [22].

3.6.5 ECVAR

ECVaR was established by Rho-Works, Advanced Analytical Systems, which is an academic software and ideal for educational and individual use. With this program asset prices can be loaded easily from XLS or CSV files and VAR, CVaR, Beta VaR, Optimum VaR, backtesting measures and more for a simple weighted investment portfolios can be easily calculated. It is also use a full-valuation historical-simulation approach on the VaR and other indicators estimations. There are several examples as ECVaR's customers: Université de Luxembourg and Princeton University use ECVaR on education and researches, Deloitte Research on researches, Deutsche Bank and Intesa on financial services, Adelphia on communications [23].

4. CONDITIONAL VALUE AT RISK (CVaR)

Conditional Value at Risk focuses on the extreme events with extreme risk, it calculates the risk beyond VaR, and so CVaR must be always equal or greater than VaR [3].

4.1 What is CVaR?

The term conditional value at risk was introduced by Rockafeller and Uryasev [13].

Definition 4.1: CVaR

Mathematically, CVaR is derived by taking a weighted average between Value at Risk and losses exceeding the Value at Risk.

CVaR gives the answer to the following question: what is the expected loss, which exceeds VaR with disregarding losses smaller than then α confidence interval [13]. CVaR is used also in risk management but especially it is used for setting risk concentration limits and developing investment and credit policy, because of its representation of extreme loss. Like in Value at Risk's calculation, in Conditional Value at Risk's calculation can be used also in parametric and non-parametric methodologies, but equity price movements with parametric distributions is the most widely used approach among financial institutions and banks [3].

According to Artzner, Conditional Value at Risk is also called Mean Excess Loss, Mean Shortfall or Tail Conditional Loss. Generally only one risk measurement technique is not sufficient with its all aspects. Because of this reason risk manager doesn't depend only one method. They try to eliminate the disadvantages of used method with other supporting methods. According to Artzner, CVaR is a supporting risk measure to VaR, with its significant advantages versus VaR and it is suggested to decrease its disadvantages [7]. It is used as an excellent tool in risk management and in portfolio optimization. CVaR is a consistent methodology with mean-variance method under the assumption of normality [26].

Although basically we try to minimize the CVaR figure, according to the definition of Conditional Value at Risk, portfolios with low CVaR figure must have low VaR figure as well, because of the condition of CVaR ≥ VaR [24].

According to Rockafeller and Uryasev Conditional Value at Risk method has better results on portfolio optimization than Value at Risk [25]. This is because of the convexity of CVaR and non-convexity of VaR. For better and efficient CVaR calculations stability of the data is important. In addition to this choosing of appropriate backtesting procedures is also important on efficiency of CVaR figure. It is not easy to satisfying all necessary conditions of the CVaR. So, although it is widely known that VaR has several disadvantages, it is still preferred to use because of its simplicity on apply [7].

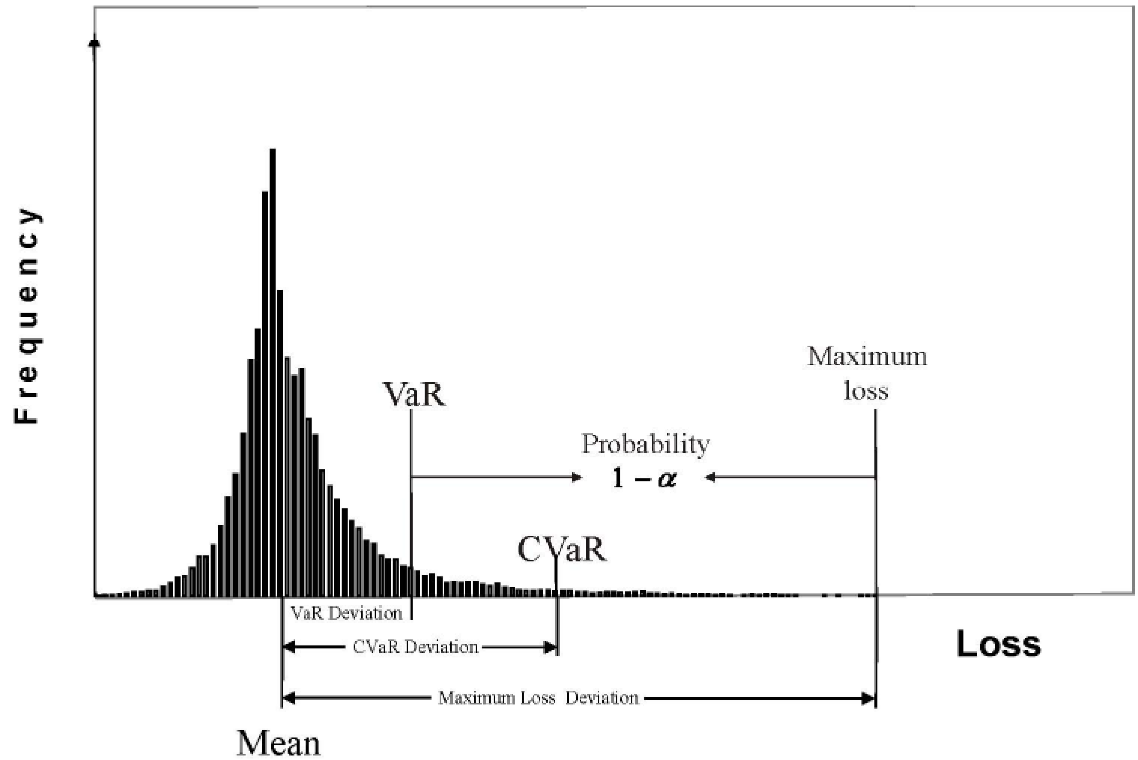

Figure 4.1: Conditional Value at Risk and Value at Risk in the same Figure

4.2 CVaR as a Coherent Metric

Despite the popularity of VaR in risk measurement, it doesn't satisfy the conditions for some mathematical properties such as sub-additivity and convexity. But CVaR has not any undesirable mathematical properties such as lack of sub-additivity or convexity [24]. So, CVaR is satisfied the coherent risk metric conditions, which are monotonicity, sub-additivity, homogeneity and risk-free condition. In addition to these properties CVaR is also satisfied the convexity rule, which means it guarantees the elimination of possibility for local minimum point being different from global minimum point.

4.3 Approaches of CVaR and Their Characteristics

There are two approaches for the Conditional Value at Risk, which are known as upper CVaR ($CVaR^+$) and lower CVaR ($CVaR^-$).

4.3.1 Upper CVaR ($CVaR^+$)

Upper CVaR is also called Mean Excess Loss and Expected Shortfall and it is the expected value of X strictly exceeding VaR.

$$\mathrm{CVaR}^{+}\ (\mathrm{X})\ =\mathrm{E}\ [\mathrm{X}\mid \mathrm{X} > VaR\ (X)]$$

4.3.2 Lower CVaR ($CVaR^-$)

Lower CVaR is also called Tail VaR and it is the expected value of X weakly exceeding VaR, which means expected losses which are equal to or exceed VaR.

$$\mathrm{CVaR}^{-}\ (\mathrm{X})\ =\mathrm{E}\ [\mathrm{X}\mid \mathrm{X}\ \geq \mathrm{VaR}(\mathrm{X})]$$

4.3.3 Characteristics of CVaR's Approaches

There is a general inequality for VaR, CVaR, $CVaR^+$ and $CVaR^-$,

$$\mathrm{VaR}\ \leq\ CVaR^{-}\ \leq\ \mathrm{CVaR}\ \leq\ CVaR^{+}$$

If the loss distribution function has any jump at VaR, then the inequalities can be strict [25].

As we mentioned before, CVaR is convex, VaR, $CVaR^+$ and $CVaR^-$ may be non-convex. And $CVaR^+$ is discontinuous function for general loss distributions like VaR [27, 28].

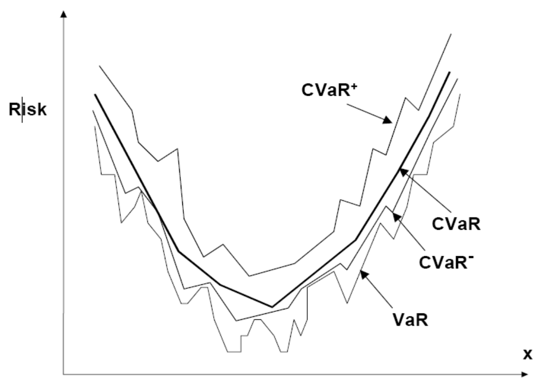

Figure 4.2: Convexity of VaR and CVaR

There is another mathematical property for CVaR calculation. CVaR is a weighted average of VaR and $CVaR^+$ and it can be formulated as follows;

$$\text{CVaR} = \alpha\, \text{VaR} + (1-\alpha)\, \text{CVaR}^+$$

This formula is interesting because of the fact that, although neither VaR nor $CVaR^+$ is coherent risk metric, coherent risk metric CVaR can be defined by non-coherent VaR and $CVaR^+$ [25].

4.4 How to Calculate CVaR?

CVaR method is developed for a risk measurement method to reduce the inconsistencies in the VaR model. It shows us that the mean of the excess distributions over the threshold VaR. It can be defined as follows;

$$\text{CVaR}(X) = E\,[-X \mid -X \geq \text{VaR}(X)]$$

The biggest criticism on CVaR is that, small losses with higher probability and higher losses with greater possibility have the same importance level for CVaR, which is different from what investors are thinking. For example, investors can prefer lower losses with higher probability. From that view for investors, who are more sensitive for higher losses CVaR may be a non-advantageous model [19].

CVaR is the mean value of the worst 1- α*100% losses [24]. For instance, if we are measuring VaR at a 95% confidence level (β=0.95), CVaR is the average of the 5% worst losses [3].

There are two approaches to calculate CVaR. The first one is parametric approach as we used it in the VaR calculation it makes sense. The second one is non-parametric approach. Non-parametric approach is used because of the limitations of parametric approach. It will yield a ranking spread for CVaR that is the same as VaR, which may not specify the extreme returns. So, non-parametric approach is used with the parametric one.

4.4.1 Parametric Approach

As in the parametric VaR calculation, there is also a normality assumption for the distribution [3]. Huang mentioned, that parametric CVaR can be calculated easily with the following formula,

$$\mathrm{CVaR} = \frac{\exp(-q^2/2)}{\alpha\sqrt{2\Pi}}\sigma$$

Where q is the tail 100α percentile of a standard normal distribution (e.g. 1.645 as obtained from standard distribution tables for 95% confidence).

4.4.2 Relative CVaR

In this approach CVaR is calculated with the actual (1-α) % worst losses. It can be calculated with the following formula,

$$CVaR_{Relative} = \mathrm{c}' \text{ x } \mathrm{V} \text{ x } \sigma$$

And c’ can be calculated with the formula below.

$$c' = PDF_{SN}(c) \ / \ CDF_{SN}(-1 * c)$$

Where PDF_{SN} and CDF_{SN} is the Probability Density Function and Cumulative Distribution Function for the Standard Normal (Normal with mean 0 and standard deviation of 1) [20].

For example, if we want to find the confidence interval for the 95% CVaR we can take the 95% VaR interval (1.64) and apply the adjustment on Microsoft Excel,

$$c' = \frac{NORMDIST\ (1.64{,}0{,}1, FALSE)}{NORMDIST\ (-1 * 1.64{,}0{,}1, TRUE)} = 2.058$$

We could then see that the 5% CVaR formula would be:

$$CVaR = 2.058 \text{ x V x } \sigma$$

5. COMPARISION of VaR and CVaR

Risk managers should take into consideration in their usage of VaR and CVaR's advantages and disadvantages. Basically, their choice is contradictive because of the following characteristics of VaR and CVaR:

- different mathematical properties
- more simple procedures in optimization
- acceptance and understanding by upper managers
- stabilization in statistical estimation [28]

DISADVANTAGES	
VaR	**CVaR**
Beyond the confidence level VaR doesn't account for distribution's properties.	It is not easy to use efficient Backtesting procedures.
For skewed distributions VaR is insufficient and may lead undesirable results.	Today there is no compatibility for CVaR with system help software and system infrastructure.
VaR is a non-convex and discontinuous for discrete distributions.	CVaR is more sensitive than VaR on error estimation.
It is hard to use VaR on alone in portfolio optimization.	Ensuring the estimation of static data is not always easy.
It is not sub-additive.	CVaR accuracy is heavily affected by accuracy of tail modeling.

Table 5.1: Disadvantages of VaR and CVaR [7, 28]

ADVANTAGES	
VaR	**CVaR**
VaR can be estimated both with parametric and non-parametric models.	It is a coherent risk measure, so it satisfies the sub-additivity rule.
It is easy to use Backtesting procedures.	It can be calculated in convex programming and also in some cases in linear programming.
VaR focuses on the part of the distribution specified by the confidence interval.	CVaR focuses on the risk beyond VaR.
Estimation procedures are stable.	CVaR is continuous with respect to confidence interval.
VaR is simpler than CVaR as a risk management concept and it has a clear interpretation.	CVaR is a convex function with respect to weights of the assets.
It is used widely as a risk measurement technique in finance on daily basis and it is compatible with system help software and system infrastructure.	It is used widely as a risk measurement technique in insurance and credit management.

Table 5.2: Advantages of VaR and CVaR [7, 28]

6. APPLICATION FOR VaR and CVaR

We have applied VaR and CVaR methods to ISE 100 index data between the dates 04.01.1999 – 27.01.2000 and 03.01.2011 - 30.12.2011. In the following figures we can see for both of the series their own graphs, their returns and the histograms of their return.

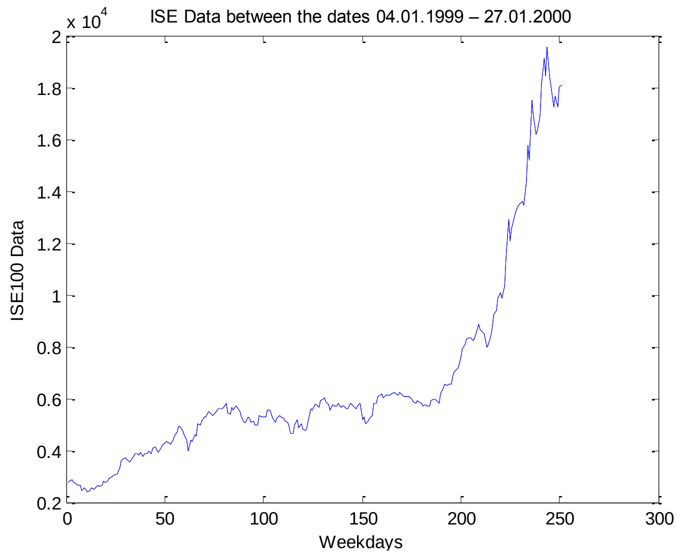

Figure 6.1: ISE 100 Data between the Dates 04.01.1999 – 27.01.2000

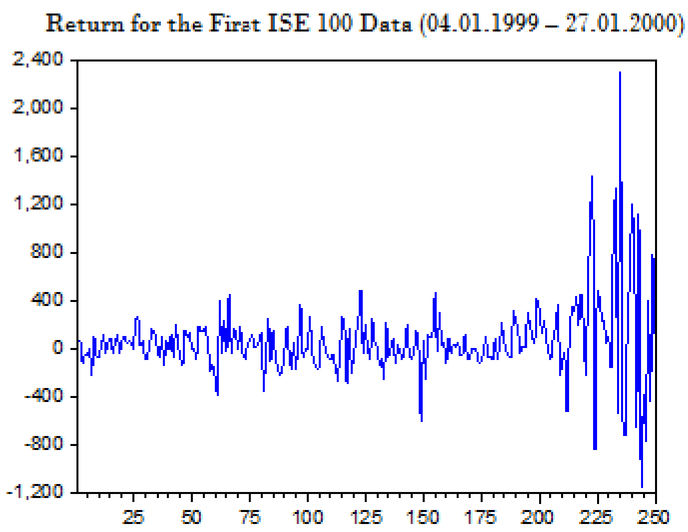

Figure 6.2: Asset's Return for the First Data

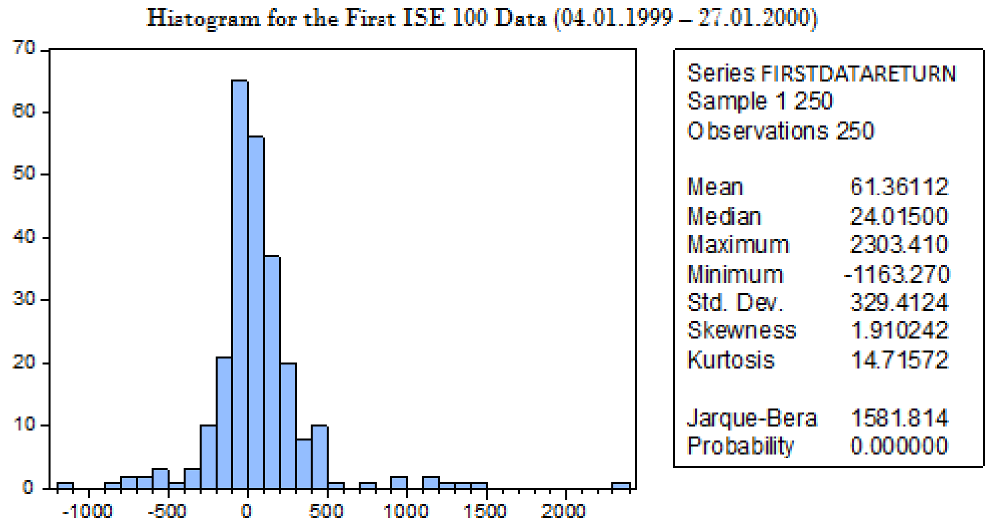

Figure 6.3: Histogram of the Return for the First Data

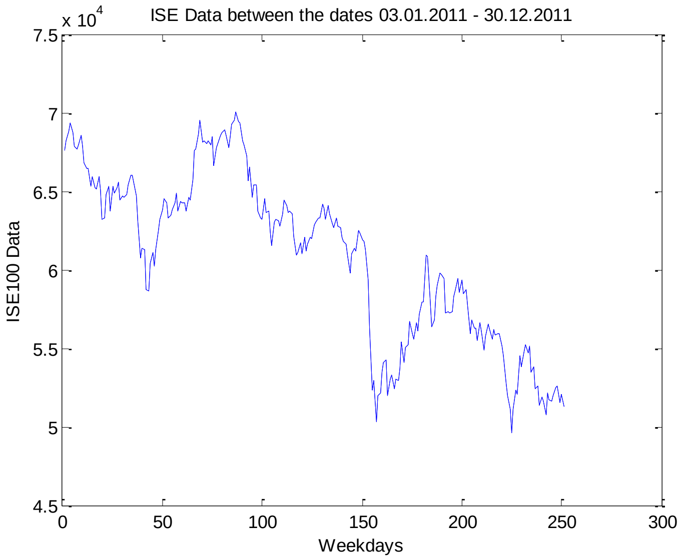

Figure 6.4: ISE 100 Data between the Dates 03.01.2011 – 30.12.2011

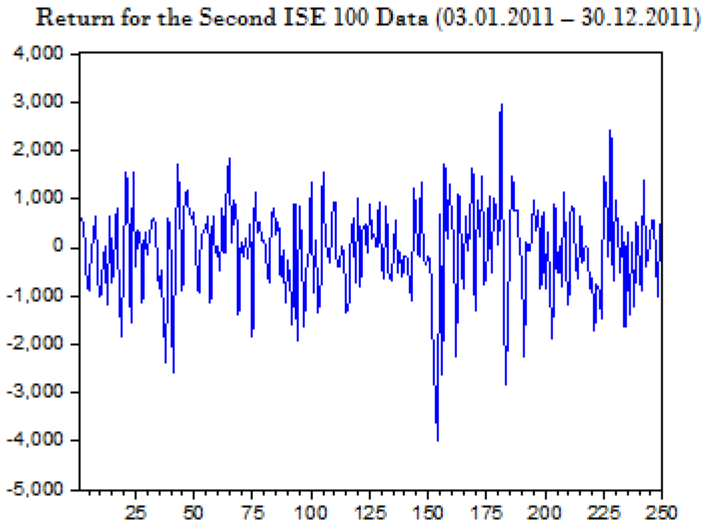

Figure 6.5: Asset's Return for the Second Data

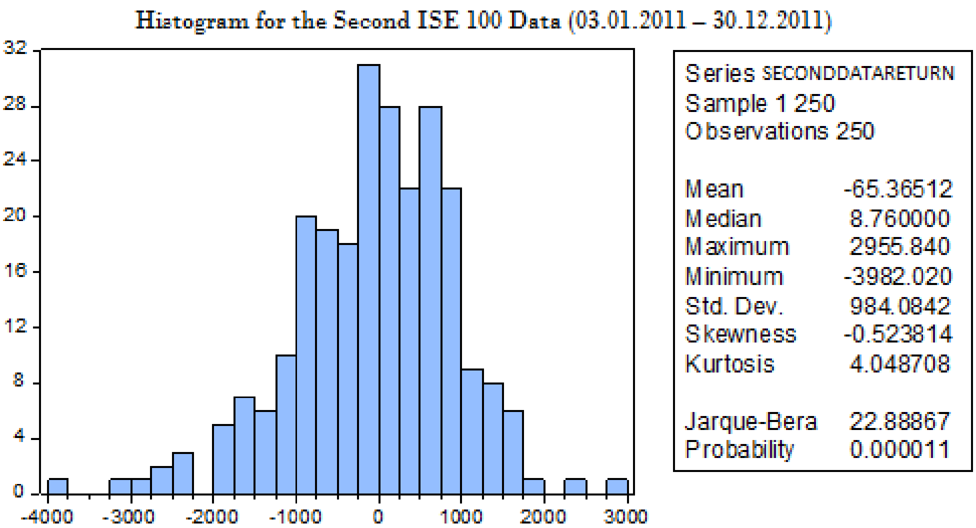

Figure 6.6: Histogram of the Return for the Second Data

As you see in Figure 6.1 our first data has an upper trend and our second data in Figure 6.4 has a downward trend. For both of the series returns are stationary about zero, as you see in Figure 6.2 and Figure 6.5. If we have taken a look on the histogram of the first return series in Figure 6.3, we can easily realize that there is a normal distribution for the return. But for the second return series' histogram in Figure 6.6 we can't mention about normality, because of the asymmetry.

- Higher positive Kurtosis value indicates a peaked distribution, when negative Kurtosis value indicates a flat distribution.
- A negative skewness value indicates that the tail on the left side of the probability density function is longer than the right side, which has the bulk of the values. And a positive skewness value indicates that the tail on the right side of the probability density function is longer than the left side, which has the bulk of the values.

If we have looked and compared the Kurtosis and Skewness values in Figure 6.3 and in Figure 6.6 with the above characteristics of these concepts, we can say that because of the higher positive Kurtosis value the first probability density function has a higher peak than second one and we can easily see the left tail on the second histogram with negative Skewness value.

6.1 Application of VaR

Then we have applied VaR calculations on ECVaR for 10 days holding period both for 99% and 95% confidence interval. The red area in the histogram is the loss below the selected confidence level.

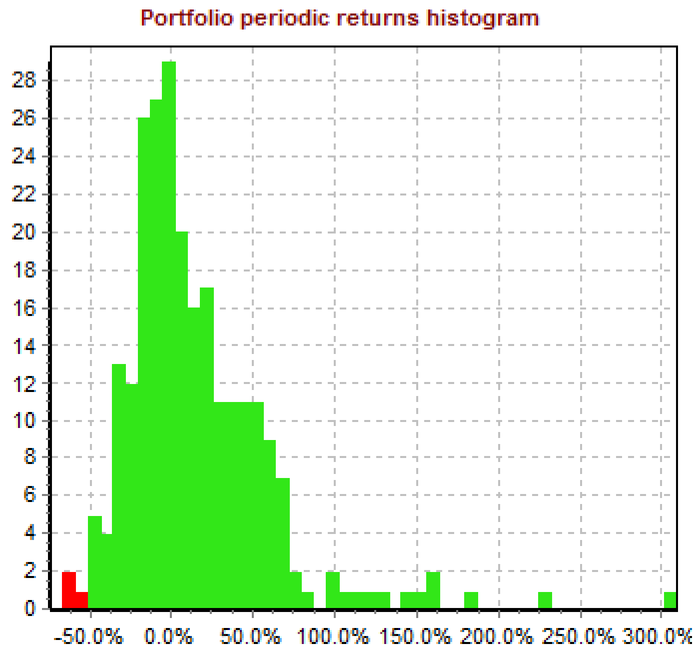

Figure 6.7: Portfolio Periodic Returns Histogram for 99% Confidence Interval for First Data

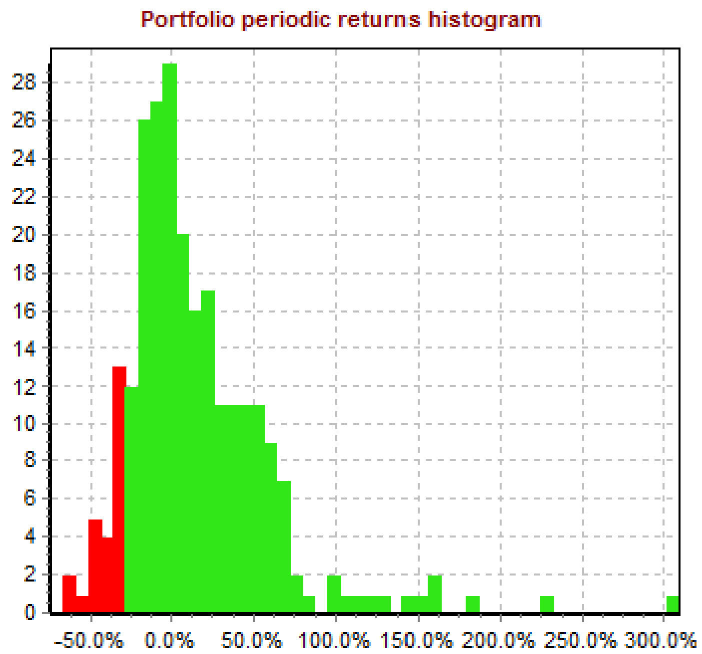

Figure 6.8: Portfolio Periodic Returns Histogram for 95% Confidence Interval for First Data

We have calculated VaR value as -10,452.79 for the 99% confidence level and -6,464.91 for the 95% confidence level. This means that our portfolio doesn't lose more than these values with the selected percent of confidence.

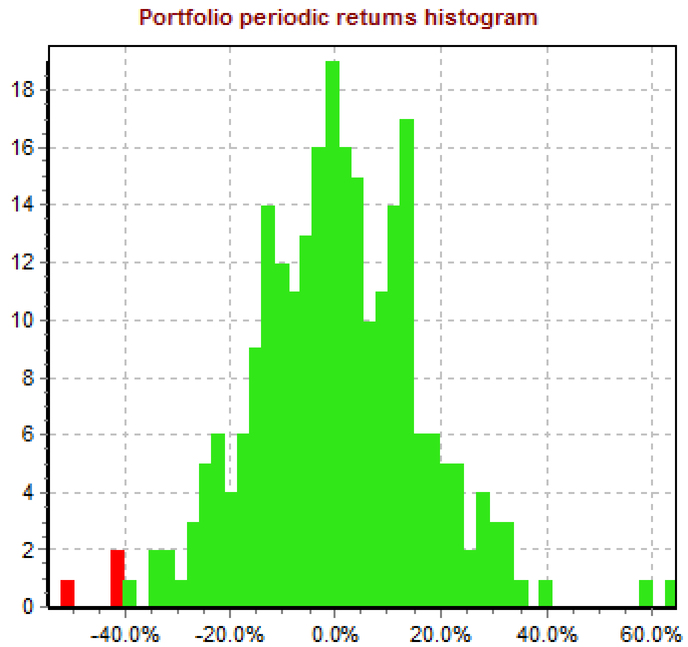

Figure 6.9: Portfolio Periodic Returns Histogram for 99% Confidence Interval for Second Data

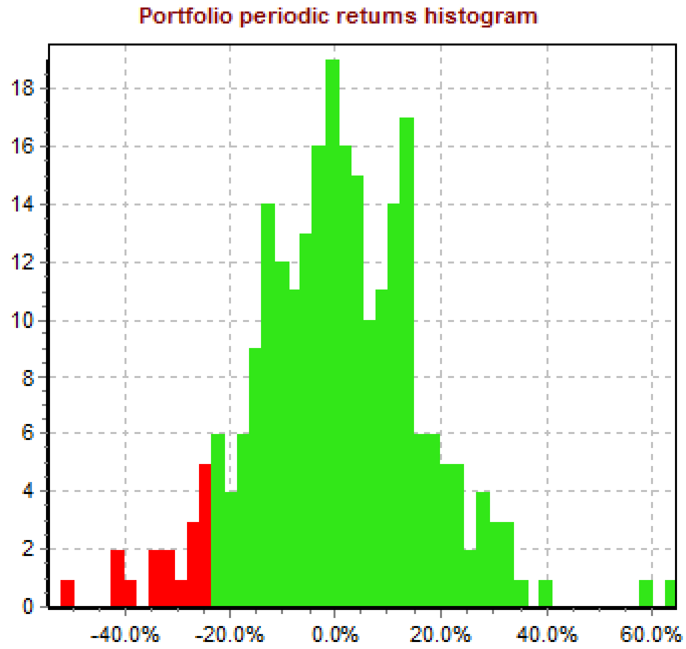

Figure 6.10: Portfolio Periodic Returns Histogram for 95% Confidence Interval for Second Data

We have calculated VaR value as -20,605.06 for the 99% confidence level and -13,190.21 for the 95% confidence level. This means that our portfolio doesn't lose more than these values with the selected percent of confidence.

We have gained the following graph in Figure 6.11 with ECVaR, where we have seen different VaR values for different time intervals for the second data series with 95% confidence interval. For this data set we can say that VaR figure increases with the increased holding period.

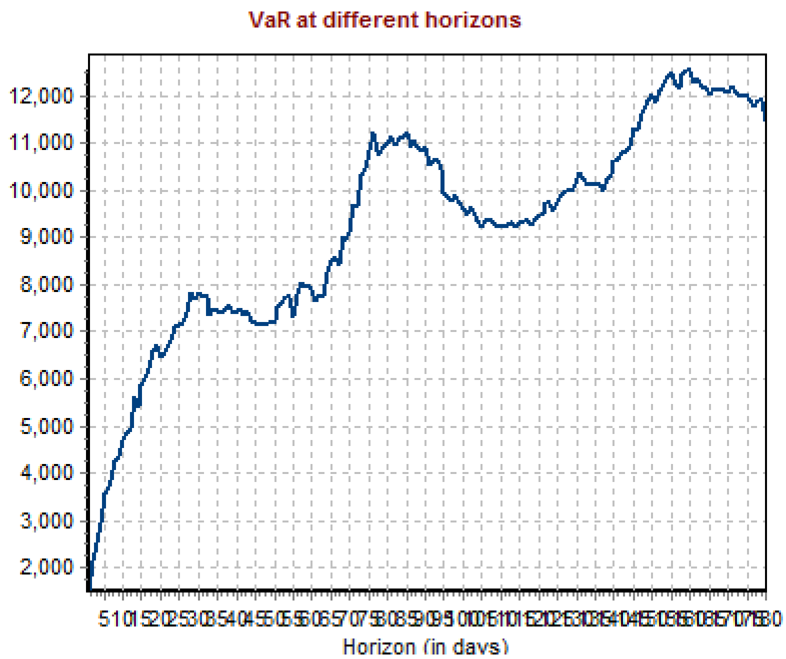

Figure 6.11: VaR Figures at Different Holding Periods

6.2 Application of CVaR

After calculation of VaR, we have applied CVaR calculations on ECVaR for 10 days holding period for couple of the series both for 99% and 95% confidence interval. The red area in the histogram is the loss below the selected confidence level as in the VaR graphs and the blue bar shows the CVaR level.

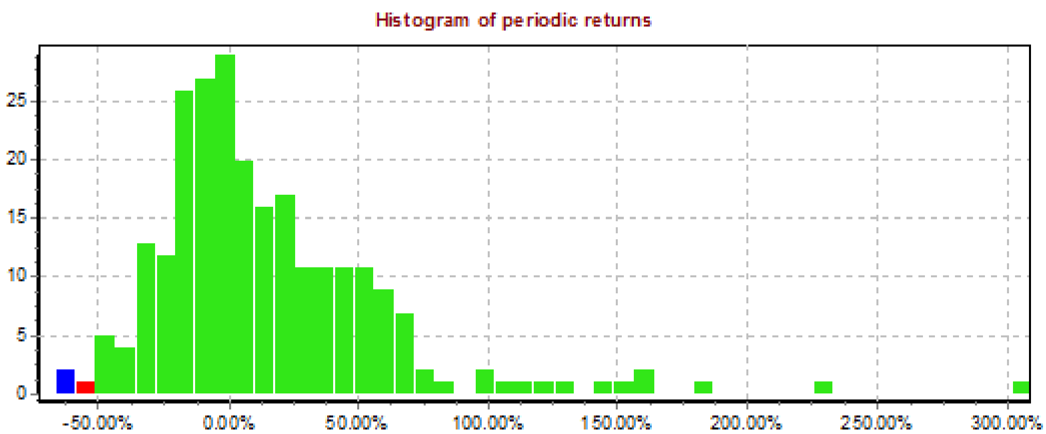

Figure 6.12: Portfolio Periodic Returns Histogram for 99% Confidence Interval for First Data

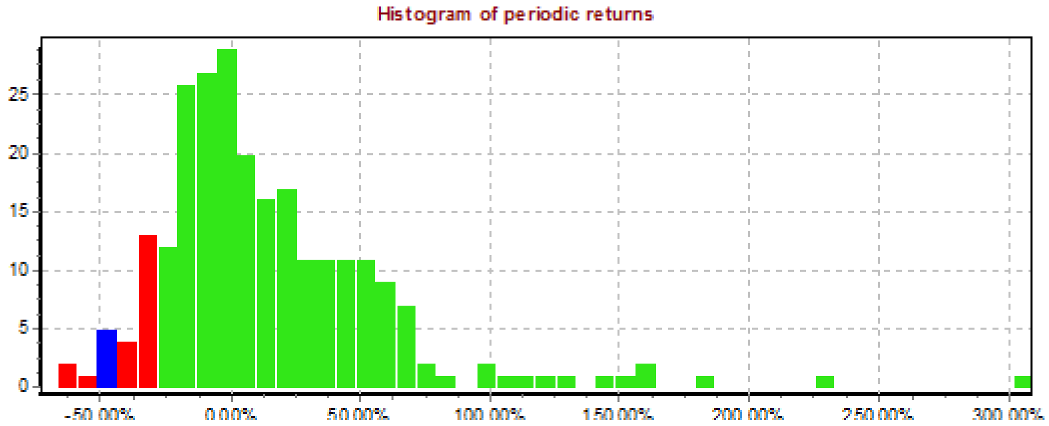

Figure 6.13: Portfolio Periodic Returns Histogram for 95% Confidence Interval for First Data

For the 99% confidence level, we have calculated CVaR value as -11,513.02, $CVaR^+$ value as -11,513.82 and $CVaR^-$ value as -11,160.14.

This means that our portfolio doesn’t lose more than -11,513.02 with 99% of confidence.

For the 95% confidence level, we have calculated CVaR value as -11,320.95, CVaR+ value as -11,401.73 and CVaR- value as -11,229.82.

This means that our portfolio doesn’t lose more than -11,320.95 with 95% of confidence.

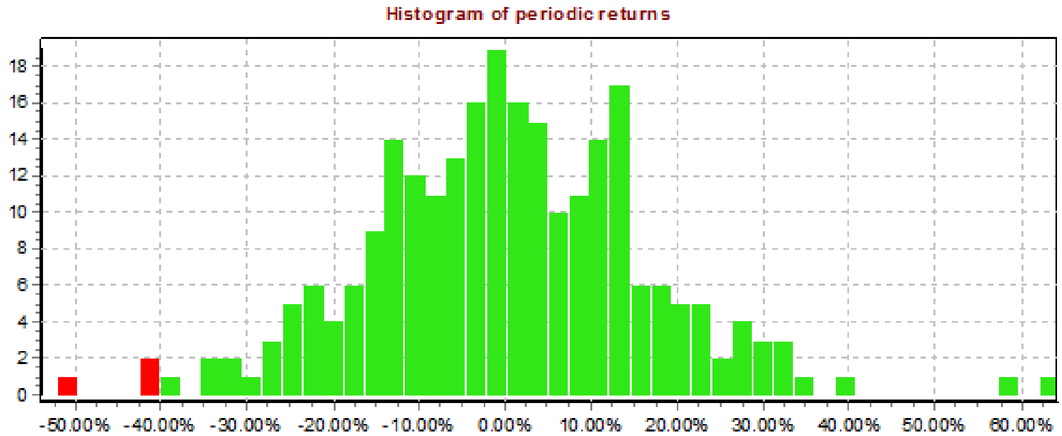

Figure 6.14: Portfolio Periodic Returns Histogram for 99% Confidence Interval for Second Data

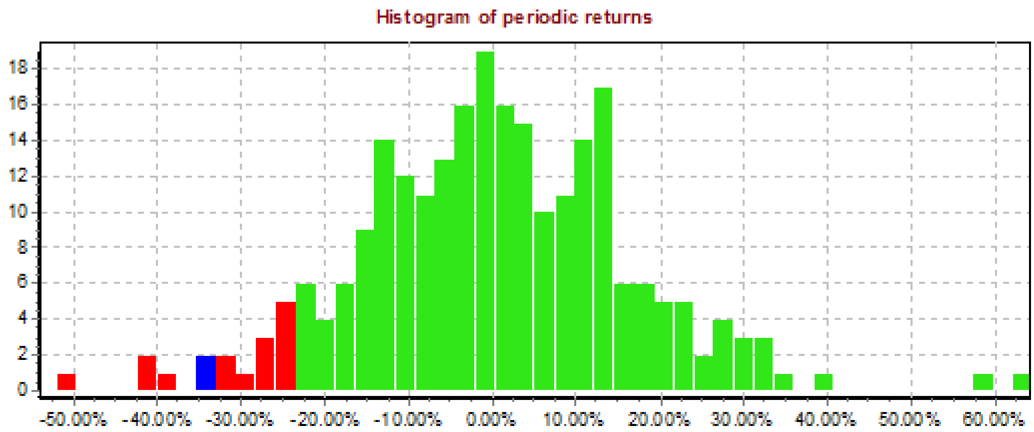

Figure 6.15: Portfolio Periodic Returns Histogram for 95% Confidence Interval for Second Data

For the 99% confidence level, we have calculated CVaR value as -22,781.72, $CVaR^+$ value as -23,870.05 and $CVaR^-$ value as -22,781.72.

This means that our portfolio doesn't lose more than -22,781.72 with 99% of confidence.

For the 95% confidence level, we have calculated CVaR value as -17,561.37, $CVaR^+$ value as –17,725.29 and $CVaR^-$ value as -17,376.44.

This means that our portfolio doesn't lose more than -17,561.37 with 95% of confidence.

If we have take a look at CVaR, $CVaR^+$, $CVaR^-$ and VaR values and compare them, we can see that the following inequality is satisfied for all of our samples.

$VaR \leq CVaR^- \leq CVaR \leq CVaR^+$

As we mentioned before on the third section CVaR is a weighted average of VaR and $CVaR^+$ and shown as $CVaR = \alpha\ VaR + (1- \alpha)\ CVaR^+$ for any $\alpha > 0$.
If we have chosen α as (1-0.9638551029), we can satisfy that property too.

7. CONCLUSION

Since risk management has an increasing significance in financial market, we aimed to create a handbook about Value at Risk and Conditional Value at Risk, which are important methods for risk measurement. For this purpose we have given theoretical information about VaR and CVaR related to their calculations, methods, coherency as risk metrics. Considering these theoretical knowledge, we have compared these methods by using ISE 100 data between the dates 04.01.1999 – 27.01.2000 and 03.01.2011 - 30.12.2011 with the programs MATLAB, Eviews and ECVaR. Then we have seen that our results are compatible with the information in literature about Value at Risk and Conditional Value at Risk. In our application we have satisfied for our sample ISE 100 data CVaR figures are greater than VaR figures and CVaR can be shown as weighted average of VaR and $CVaR^+$.

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